BIOELECTROMAGNETICS

Goal

- explain basic principles, relationships, methods of BEM
  - ↳ i.e. science of em field interaction with biological tissue
- define, give examples for most important physical phenomena, classify them, corresponding wavelength/freq.
- give overview over measurement & numerical techniques for characterization of em fields in practical applications
- give examples for therapeutic & diagnostic utilization of em fields in medical technology
- how to characterize em fields in biological tissue
- relate to & make use of Maxwell's Equations
- assess effects that Maxwell's Equations predict for biological tissue
- order effects corresponding to wavelength/freq.
- analyze effects in a quantitative way, also in therapeutic & diagnostic applications

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- physical phenomena + classification of wavelength/freq., qualitative & quantitative analysis
  - ↳ behaviour of electromagnetic fields and interaction with biological tissue
- characterization of EM fields: measurement & numerical techniques
- utilization of EM fields in therapeutic & diagnostic applications
- Maxwells equations: Understand, also applications

Units

$\vec{E}$ $\left[\frac{N}{C}=\frac{V}{m}\right]$: electric field strength

$\varepsilon_0\varepsilon_r$

$\vec{D}$ $\left[\frac{As}{m^2}=\frac{C}{m^2}\right]$: electric flux density

$\vec{H}$ $\left[\frac{A}{m}\right]$: magnetic field strength

$\mu_0\mu_r$

$\vec{B}$ $\left[\frac{N}{Am}=\frac{Vs}{m^2}=T\right]$: magnetic flux density

Q $[C=As]$: Charge

I $\left[A=\frac{C}{s}\right]$: Current

J current density

thermal power density

$p = JE$

Dimensionality	Charge Density	Current Density	Isotropic Current Source	Potentials $\Phi(\vec{r})$ of current densities (homogeneous, isotropic medium)
Volume	$\rho\left[\frac{As}{m^3}\right]$	$\vec{J}\left[\frac{A}{m^2}\right]$	$I_V(\vec{r})\left[\frac{A}{m^3}\right]$	$\frac{1}{4\pi\kappa}\cdot\iiint_\infty\frac{I_V(\vec{r})}{\lvert\vec{r}-\vec{r}\,'\rvert}dV'$ conductivity
Surface	$\sigma\left[\frac{As}{m^2}\right]$	$\vec{J}_S\left[\frac{A}{m}\right]$	$I_S(\vec{r})\left[\frac{A}{m^2}\right]$	$\frac{1}{4\pi\kappa}\cdot\iint_\infty\frac{I_S(\vec{r})}{\lvert\vec{r}-\vec{r}\,'\rvert}dA'$
Line	$\lambda\left[\frac{As}{m}\right]$	$I\cdot d\vec{r}\,[A]$	$I_L(\vec{r})\left[\frac{A}{m}\right]$	$\frac{1}{4\pi\kappa}\cdot\int_\infty\frac{I_L(\vec{r}\,')}{\lvert\vec{r}-\vec{r}\,'\rvert}dr'$
Single Element			$I_{0,\vec{r}}$ $[A]$ ↳ Point source in $\vec{r}$	$\frac{I_{0,\vec{r}}}{4\pi\kappa}\cdot\frac{1}{\lvert\vec{r}-\vec{r}\,'\rvert}$

EM Fields

- around power lines: $1\frac{kV}{m}$, $3\mu T$
- around earth: $100\frac{V}{m}$, $50\mu T$
- max permissible Peak field values: $0.083\frac{kV}{m}$, $83\frac{V}{m}$, $27\mu T$
  - ↳ 3kHz to 10MHz
- max permissible avg field values
  - @ 10-400 MHz: $28\frac{V}{m}$, $0.073\frac{A}{m}$
  - @ 400 MHz-2GHz: $28\text{-}61\frac{V}{m}$, $0.073\text{-}0.16\frac{A}{m}$

Frequency Spectrum

DC, Low Freq. Range

0 Hz - 1 MHz f

0 — ∞ m … 300m λ

↑ AC power 50Hz

- Static fields, fields of stationary currents
- Model: Equivalent Circuits

High Freq. Range

$c=\lambda\cdot f \Rightarrow \lambda=\frac{c}{f}=\frac{3\cdot10^8}{10^9}$

1GHz: 32cm

1MHz - 10THz

300m … $3\cdot10^{-5}m = 30\mu m$

↑ cell phones 2GHz

- quasi-stationary fields, fully dynamic fields
- Model: Plane Waves

IR & Optical Freq. Range

10THz – 2PHz

30µm … 0.15 µm

↑ Green Light 600THz

visible light: 740nm … 380nm 400THz – 790THz

- fields as light rays geometrical optics
- Model: Lightrays

Ionization Freq. Range

> 2PHz

0.15µm ….

↑ X-rays (1000PHz)

ab 2PHz: Ionisation von Wasserstoff H bei 8eV

- fields as photons
- Model: Photons

Electromagnetics Basics

Capacitors

$$\underset{\text{Total charge}}{Q} = \underset{\text{number of electron}}{n}\cdot\underset{\text{charge of one electron}}{e}$$

$e \approx 1.602\cdot10^{-19}C$

→ quantized!

$$\underset{\text{total charge}}{Q} = \iiint_V \underset{\text{charge density}}{\rho}\, dV$$

$$= \oiint \vec{D}\cdot d\vec{A}$$

↳ electric flux density

$$\underset{\text{Force}}{\vec{F}_e} = \underset{\text{total charge}}{Q}\cdot\underset{\text{electric field}}{\vec{E}}$$

Q positive: F same direction as E

Q negative: F opposite direction than E

$$\varphi_B = \int_{-B}^{B}\vec{E}\cdot d\vec{s}$$

$$\underset{\text{Voltage}}{U} = \underset{\text{electrical field}}{E}\cdot\underset{\text{distance}}{d} \Leftrightarrow E=\frac{U}{d}, \text{ in } \frac{V}{m}$$

$\frac{1}{c}\sim E$

$$\underset{\text{Total charge}}{Q} = \underset{\text{Capacitance}}{C}\cdot\underset{\text{Voltage}}{U} \Rightarrow U=\frac{Q}{C},\ C=\frac{Q}{U}$$

$$C=\varepsilon_0\cdot\varepsilon_r\cdot\frac{A}{d}$$

$$\underset{\text{Energy}}{W_e} = \frac{1}{2}\cdot\underset{\text{capacitance}}{C}\cdot U^2$$

← Energy stored in capacitor

$$\underset{\text{inducted voltage on coil}}{U_{ind}} = -\frac{d\Phi}{dt}$$

change of magnetic flux

Faradays law: Magnetic flux inducting voltage to one coil ring of wire

$$= -N\cdot\frac{d\Phi}{dt}$$

↳ N being count of coil turns

$$c=\lambda\cdot f \qquad c=\frac{1}{\sqrt{\mu_0\cdot\mu_r\cdot\varepsilon_0\cdot\varepsilon_r}}$$

$c = 2.998\cdot10^8\frac{m}{s}$

$\varepsilon_0 \approx 8.854\cdot10^{-12}\frac{F}{m}$ $\left[\frac{As}{Vm}\right]$

$\mu_0 = 4\cdot\pi\cdot10^{-7}\frac{H}{m}$ $\qquad B = H\cdot\mu_0$

Coils/Inductors

$$\underset{\text{current}}{I} = \frac{dQ}{dt}$$

← charges flowing per time unit

$$= \iint\vec{J}\cdot d\vec{A}$$

↳ current density

$$I_{displacement} = \iint_A\frac{d\vec{D}}{dt}d\vec{A}$$

↳ displacement current field

$$I_{total} = \int_A\left(\sigma\vec{E}+\varepsilon\frac{\partial\vec{E}}{\partial t}\right)\cdot d\vec{A}$$

Magnetic Force on current carrying conductor

$$F_m = \underset{\text{current}}{I}\cdot\underset{\text{magnetic flux density perpendicular to current}}{B_\perp}\cdot\underset{\text{length of conductor}}{l}$$

$$\vec{F} = \iiint_V\vec{f}\,dV$$

← Force density

$$\underset{\text{Magnetic flux}}{\Phi} = \underset{\text{magnetic flux density perpendicular}}{B_\perp}\cdot\underset{\text{Area (?)}}{A}$$

$$= \iint_A\vec{B}\cdot d\vec{A}$$

$$\underset{\text{Magnetic flux}}{\Phi} = \underset{\text{Inductance}}{L}\cdot\underset{\text{current}}{I}$$

$$\underset{\text{Energy stored in coil}}{W_m} = \frac{1}{2}\cdot L\cdot I^2$$

$\vec{D}=\varepsilon_0\varepsilon_r\cdot\vec{E}$

$\vec{B}=\mu_0\mu_r\cdot\vec{H}$

Monopole

$\vec{E}=\frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\sim\frac{1}{r^2}$

↳ $\vec{E}$-field decays quadratically

Dipole: Decay $\frac{1}{r^3}$

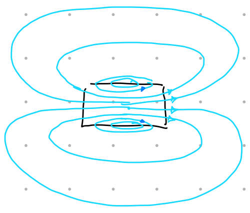
Infinite Plane: const! no decay!

$\vec{J}_{conduction} = \kappa\cdot\vec{E}$ ← Ohm's law in 3D

$\vec{J}_D = \varepsilon_0\cdot\frac{\delta E}{\delta t}+\frac{\delta P}{\delta t}$ ← polarization of medium

$\vec{D}=\varepsilon_0\vec{E}+\vec{P}$ $\qquad \vec{J}_{total} = \vec{J}_{conduction}+\vec{J}_{Displacement}$

No magnetic monopoles

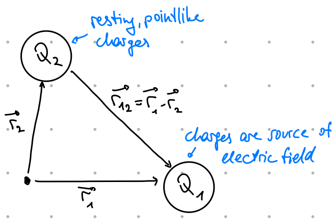


Lorentz force

$$\vec{F}_L = q\cdot(\vec{E}+\underset{\text{velocity}}{\vec{v}}\times\vec{B})$$

Rechte Handregel

Coulomb's Law



resting, pointlike charges

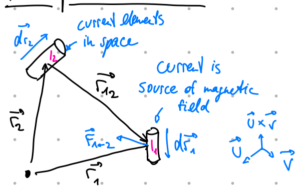
charges are source of electric field

$$\vec{F}_{1\leftarrow2} = \frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r}\cdot\frac{\vec{r}_{12}}{r_{12}^3} = -\vec{F}_{2\leftarrow1}$$

$$\vec{F} = q\cdot\vec{E}$$

← Probeladung

Ampère's Law



current elements in space

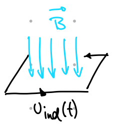
current is source of magnetic field

Rechte Hand - Regel

$$d\vec{F}_{1\leftarrow2} = \frac{\mu_0\mu_r\cdot I_1\cdot I_2}{4\pi}\cdot\frac{d\vec{r}_1\times(d\vec{r}_2\times\vec{r}_{12})}{r_{12}^3}$$

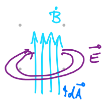
$$= -d\vec{F}_{2\leftarrow1}$$

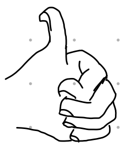
Faraday's Law



$$U_{ind}(t) = \oint\vec{E}\cdot d\vec{r} = -\underset{\text{magnetic flux}}{\frac{d\Phi}{dt}} = -\frac{d}{dt}\iint\underset{\text{Magnetic flux density}}{\vec{B}}\cdot d\vec{A}$$

↑ voltage is generated by changing magnetic fields





Linke Hand Regel

Maxwells Equations

Gauss's Law for Magnetism

$$\nabla\cdot\vec{B} = 0$$

$$\oiint\vec{B}\,d\vec{A}=0$$

Magnetic fields do not possess magnetic charges ("monopoles" as sources. ⇒ B-field lines have no start & no end.

H-field lines may have start & end

Coulomb's Law (/ Gauss's Law)

$$\nabla\cdot\underset{\text{displacement current}}{\vec{D}} = \rho \leftarrow \text{charge density}$$

$$\oiint\vec{D}\,d\vec{A} = Q \leftarrow \text{total charge}$$

$$= \varepsilon_0\varepsilon_r\cdot\oiint\vec{E}\,d\vec{A}$$

electrostatic field

electric charges are sources of electric fields. Electric field lines start at positive charges and end at negative charges.

$\frac{dE(x)}{dx}\cdot\varepsilon = \rho(x)$

current density + changing displacement current induce curl in magnetic field

$$\int_A\underbrace{\vec{D}\cdot d\vec{A}}_{\text{Skalarprodukt}} = \psi \leftarrow \text{elektrischer Fluss } [As]$$

$\Phi = \frac{\psi}{\varepsilon_0}$

← Physikalische Definition des elektrischen Flusses

Maxwell-Ampère's Law

$$\nabla\times\vec{H} = \underset{\text{current density}}{\vec{J}}+\frac{\partial\vec{D}}{\partial t}$$

or

$$\nabla\times\vec{B} = \mu_0\cdot\vec{J}+\mu_0\varepsilon_0\cdot\frac{\partial\vec{E}}{\partial t}$$

elektrischer Leitstrom + Magnetisierungsstrom

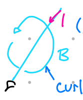
electric fields & currents are source of magnetic fields. Magnetic fields may be generated around electric currents or time-varying fields

Magnetic circulation on closed contour C is equal to electric current + time-derivative of electric flux of D-field for open surface A

$$\oint_C H\cdot d\vec{r} = \underset{\text{current}}{I}+\iint\frac{\partial}{\partial t}\vec{D}\,d\vec{A}$$

→ permanent magnet possible here

$$\oint_C\vec{B}\cdot d\vec{r} = \mu_0\mu_r\iint_A\vec{J}\cdot d\vec{A}$$



For line currents: $\oint_C\vec{B}\,d\vec{r} = \mu_0\mu_r\cdot\sum_{\text{within } C} I$

Faraday's Law

$$\underbrace{\nabla\times}_{\text{curl / "rotation"}}\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

magnitude of E-field circulation

time-derivative of magnetic flux density

Electric fields are generated by time-varying magnetic fields. (or if one moves in magnetic field that varies in space)

changing magnetic flux density induces curl in electric field (also voltage if wire through electric field)

E-field around closed contour C

Lenz rule (negative feedback)

$$\underset{\text{induced}}{U_{ind}} = \oint_C\vec{E}\,d\vec{r} = -\frac{d}{dt}\iint_A\vec{B}\,d\vec{A} = -\frac{d\Phi}{dt}$$

A → open surface A that is bounded by C

Lenz's rule: induced voltage by changing magnetic flux opposes original change in flux so current in contour gives rise to a flux

↳ negative feedback

Electrical Properties of Tissues

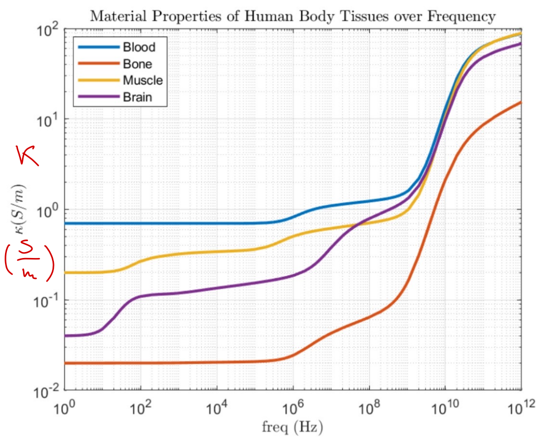
@DC - 1 MHz

Tissue Properties (Permittivity)

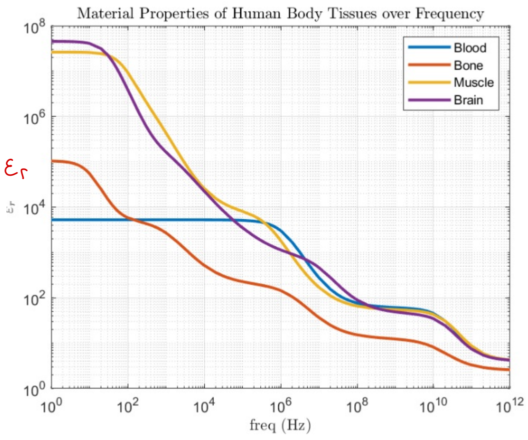
	Dielectric Const.	Conductivity ($\frac{S}{m}$)	Resistivity	Conductivity < 1MHz
Air	1	0		$10^{-15} \frac{S}{m}$
Fat	11	0.17	2060-2720 Ωcm	0.02-0.05 $\frac{S}{m}$
Bone	21	0.33	16 600 Ωcm	0.005-0.06 $\frac{S}{m}$
Skin	35	0.6		
Sceletal-Muscle Heart-Muscle	50	1.08	160 Ωcm longitudinal, 1800 Ωcm across 160-575 Ω longitudinal 424-5181 Ω across	0.05-0.4 <10kHz 0.6 @1MHz
Blood	55	1.86	150 Ωcm	0.7
Eye	67	1.97		

$1\,\Omega cm = 0.01\,\Omega m$

Conductivity



Permittivity



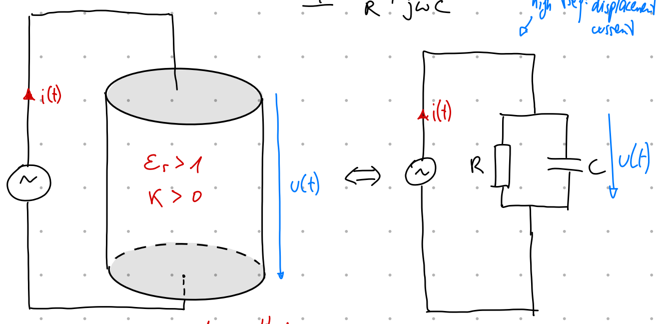
Modelling conducting dielectrics

DC to Moderate Frequencies

$\underline{\kappa}_{equiv} = \kappa + j\omega\varepsilon_0\varepsilon_r$

$\underline{Y} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega C}}$

low freq: conduction current
high freq: displacement current



current is leading voltage

$i_c(t) = C \cdot \frac{du_c(t)}{dt}$

$\underline{I} = j\omega C \cdot \underline{U}$

gleich große Ströme in R & C, wenn:

$\omega = \frac{1}{RC}$

$\underline{\varepsilon}_{equiv} = \varepsilon'_{eq} - j\varepsilon''_{eq}$ $\quad \underline{\kappa}_{eq} = j\omega\varepsilon_0\underline{\varepsilon}_{eq}$

$\kappa'_{eq} = \omega \cdot \varepsilon_0 \cdot \varepsilon''_r$

$\kappa''_{eq} = \omega\varepsilon_0\varepsilon'_r$

R & C together!!!

$R \cdot C = \frac{\varepsilon_0\varepsilon_r}{\kappa} = \tau$ ← relaxation time

Homogeneous conducting dielectrics

with freq. independent material parameters

- show both polarization and conduction
- under DC: only conduction current, polarization only at surface
- under AC: also displacement current (due to time-varying polarization)

$\frac{J_{disp}}{J_{cond}} = \omega \cdot \frac{\varepsilon_0 \cdot \varepsilon_r}{\kappa} = \omega \cdot \tau$ ← rises linearly with frequency

Complex Fields and Amplitudes

↳ to simplify calculations

$\underline{\vec{E}}(\vec{r}, t) = \underline{\hat{\vec{E}}}(\vec{r}) \cdot \exp(+j\omega t)$

-jωt can also be found

complex field ↑ $\quad$ complex amplitude of field or vector phasor

$E(\vec{r}, t) = Re[\underline{\vec{E}}(\vec{r}, t)]$ = instantaneous field

Phase Factors

$$\underline{\hat{\vec{E}}}(\vec{r}) = \begin{pmatrix} \underline{\hat{E}}_x(\vec{r}) \\ \underline{\hat{E}}_y(\vec{r}) \\ \underline{\hat{E}}_z(\vec{r}) \end{pmatrix} = \begin{pmatrix} \hat{E}_x(\vec{r}) \cdot e^{j\varphi_x(\vec{r})} \\ \hat{E}_y(\vec{r}) \cdot e^{j\varphi_y(\vec{r})} \\ \hat{E}_z(\vec{r}) \cdot e^{j\varphi_z(\vec{r})} \end{pmatrix}$$

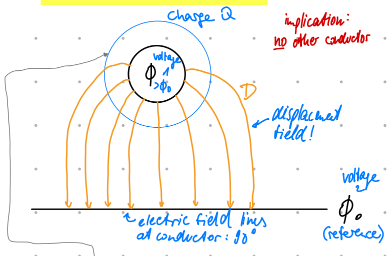
Magnitude & Time Derivative

$$\frac{\partial \underline{\vec{E}}}{\partial t} = \underline{\vec{E}}(\vec{r}) \cdot \frac{\partial}{\partial t}\left(e^{j\omega t}\right) = \underline{\vec{E}}(\vec{r}) \cdot j\omega \cdot e^{j\omega t} = j\omega \cdot \underline{\vec{E}}$$

$$|\underline{\hat{\vec{E}}}(\vec{r})| = \sqrt{\underline{\hat{\vec{E}}}(\vec{r}) \cdot \underline{\hat{\vec{E}}}^*(\vec{r})} = \sqrt{\hat{E}_x^2(\vec{r}) + \hat{E}_y^2(\vec{r}) + \hat{E}_z^2(\vec{r})}$$

Capacitance

(electrostatic field)



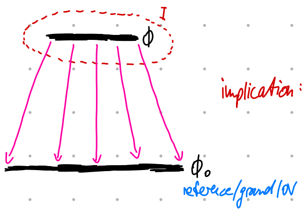
$$C = \frac{Q}{U} = \frac{\oiint \vec{D} \cdot d\vec{A}}{\phi - \phi_0} = \frac{\varepsilon_0\varepsilon_r \cdot \oiint \vec{E}\, d\vec{A}}{\phi - \phi_0}$$

surface around conductor

electrostatic potential difference

Resistance

(field of stationary current)



implication: no other electrode

$$R = \frac{U}{I} = \frac{\phi - \phi_0}{\oiint \vec{J} \cdot d\vec{A}} = \frac{\phi - \phi_0}{\kappa \cdot \oiint \vec{E} \cdot d\vec{A}}$$

surface around conductor $\quad$ ↖ conduction

$R = \frac{1}{\kappa} \cdot \frac{d}{A} \qquad RC = \frac{\varepsilon}{\kappa}$

If AC 230V applied to human hands: ≈ 95 W!

if DC 100V applied: 18 W

Complex Maxwell´s Equations

$\oiint \underline{\vec{D}} \cdot d\vec{A} = Q \qquad \oiint \underline{\vec{B}} \cdot d\vec{A} = 0$

$\oint \underline{\vec{E}}\, d\vec{r} = -\frac{d}{dt}\iint \underline{\vec{B}}\, d\vec{A} \qquad \oint_C \underline{\vec{H}}\, d\vec{r} = \underline{I} + \frac{\partial}{\partial t}\iint_A \underline{\vec{D}}\, d\vec{A}$

$\underline{\vec{D}} = \varepsilon_0\varepsilon_r(\omega) \cdot \underline{\vec{E}} \qquad \underline{\vec{B}} = \mu_0\mu_r(\omega) \cdot \underline{\vec{H}} \qquad \underline{\vec{J}} = \kappa(\omega) \cdot \underline{\vec{E}}$

Complex material parameters

$\underline{\varepsilon}_r(\omega) = \varepsilon'_r(\omega) - j \cdot \varepsilon''_r(\omega)$

$\underline{\mu}_r(\omega) = \mu'_r(\omega) - j\mu''_r(\omega)$

$\underline{\kappa}(\omega) = \kappa'(\omega) + j \cdot \kappa''(\omega)$

Imaginary parts reflect phase difference between cause & effect

$\underline{\kappa}_{equiv} = \kappa'_{equiv} + j \cdot \kappa''_{equiv} = \omega \cdot \varepsilon''_{equiv} + j\omega \cdot \varepsilon'_{equiv}$

$\underline{\varepsilon}_{equiv} = \varepsilon'_{equiv} - j \cdot \varepsilon''_{equiv} = \frac{1}{\omega}\kappa''_{equiv} - j\frac{1}{\omega}\kappa'_{equiv}$

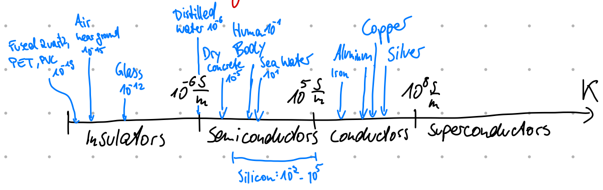
$\underline{\kappa}_{equiv} = j\omega \cdot \underline{\varepsilon}_{equiv}$

Conductivity in ideal conductors

- ideal conductor: material that responds to external electric field with electric conduction
- electric conduction: electric charge carriers of material move with exteral field

  ↳ in metals : carriers are negatively charged electrons
- electric conductivity: $\kappa = \frac{J_{conduction}}{E}$ ← often also σ

⇒ charge movement would not lead to charging the conductor itself which would generate internal E-field



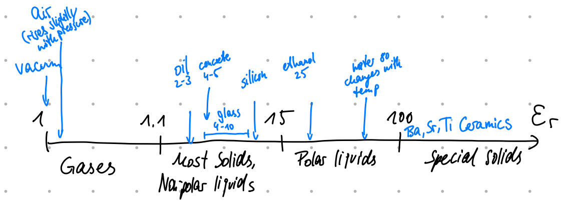
Permittivity of ideal dielectrics

- ideal dielectric: material that responds to external electric field with electric polarization
- elementary electric dipols in material are aligned with external field
- internal electric field of dipoles reduces the external field

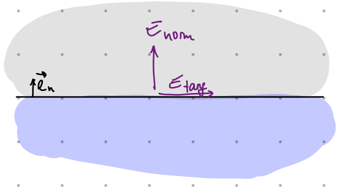
  ↳ results in smaller field of dielectric
- effect: relative permittivity / dielectric constant

$$\varepsilon_r = \frac{E_{vacuum}}{E_{dielectric}}$$

- dipoles don't move, also no movement of free charge carriers
- no conduction current
- AC current: alternating alignment leads to displacement current



Electric field boundary conditions



$\underline{E}_{tang_1} = \underline{E}_{tang_2}$

$(\kappa_1 + j\omega\varepsilon_0\varepsilon_{r1}) \cdot \underline{E}_{norm1} = (\kappa_2 + j\omega\varepsilon_0\varepsilon_{r2}) \cdot \underline{E}_{norm2}$

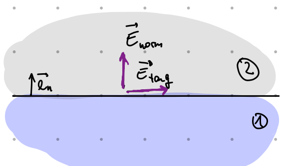
$\Rightarrow \underline{E}_{norm2} = \frac{\kappa_1 + j\omega\varepsilon_0\varepsilon_{r1}}{\kappa_2 + j\omega\varepsilon_0\varepsilon_{r2}} \cdot \underline{E}_{norm1}$

E-field: "Big change" in field distribution

External E-field ⊥ to human body

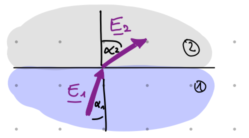
barely penetrating

Boundary for E-fields

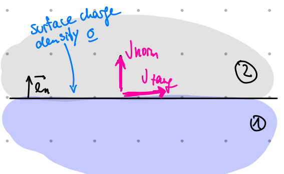


$\underline{E}_{tang_1} = \underline{E}_{tang_2}$

$\underline{\kappa}_{equiv1} \cdot \underline{E}_{norm1} = \underline{\kappa}_{equiv2} \cdot \underline{E}_{norm2}$

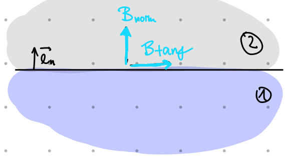


$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{\underline{\kappa}_{equiv1}}{\underline{\kappa}_{equiv2}}$



$\frac{\underline{J}_{tang_1}}{\underline{\kappa}_1} = \frac{\underline{J}_{tang_2}}{\kappa_2}$

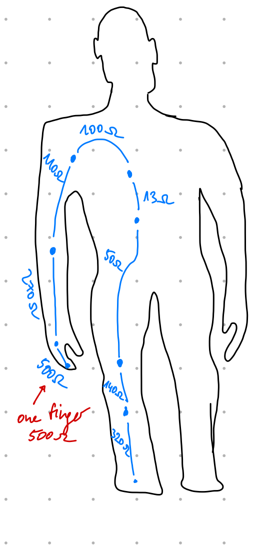
$\underline{J}_{norm1} = \underline{J}_{norm2} + j \cdot \omega\underline{\sigma}$



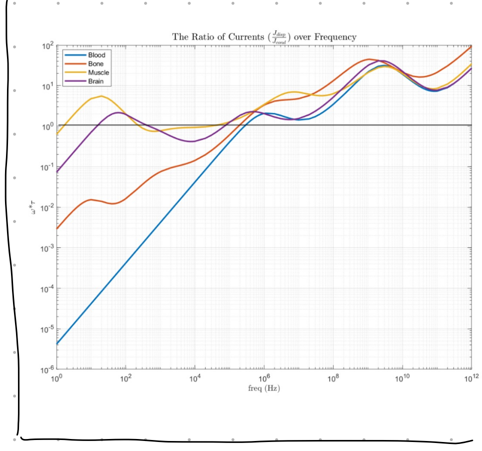
$\frac{\underline{B}_{tang1}}{\mu_{r1}} = \frac{\underline{B}_{tang2}}{\mu_{r2}}$

$\underline{B}_{norm1} = \underline{B}_{norm2}$

DC Resistance in body



Current ratio vs frequency

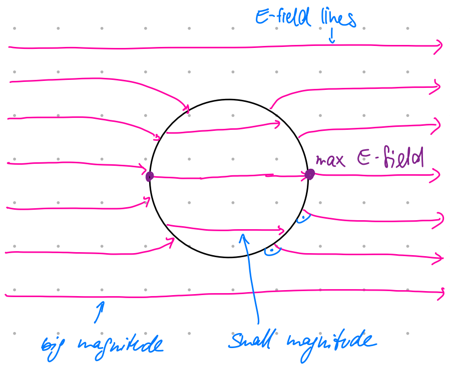


3 big reactions of matter

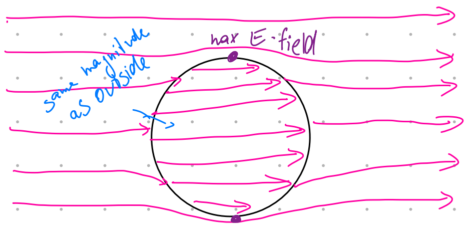
- el. polarization
- el. conduction
- magnetization

Sphere in E-field

$\varepsilon_{ext} < \varepsilon_{int}$

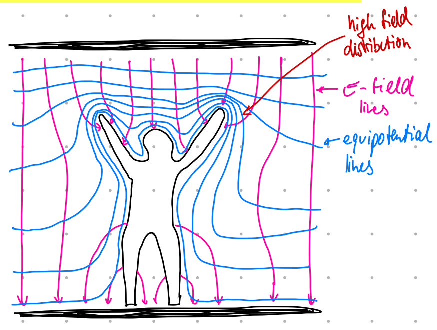


$\varepsilon_{ext} > \varepsilon_{int}$ $\quad$ max E-field strength: 3 · Eext if $\varepsilon_{int} \approx \infty$



if $\varepsilon_{ext} = \varepsilon_{int}$: invisible!

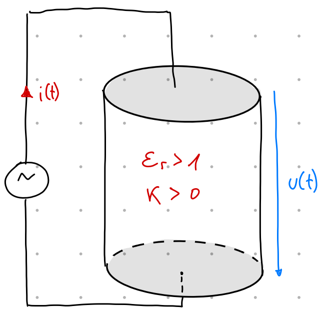
Human in 50Hz E-field



Properties of electrolytic solutions

lecture 6

Body as electrolytic solution



Body : Solution of electrolytes in water

Electrolyte: substance that ionizes when dissolved in a solvent like water (e.g. salts)

Molar concentration

Electrolytic ion concentration usually given in terms of moles, i.e. molar concentration or molality

$$C = \frac{N_{solute}}{V_{solution}} \quad [\overset{Molar}{M} = \frac{mol}{L}]$$, also given in %, ppm, ppb

$1\ mol \approx 6.022 \cdot 10^{23}$

Example: 0.9% NaCl solution in 1l water:

1l of water corresponds to 1000g.

$\underbrace{\text{salt mass} = 0.9\% \cdot 1000g}_{\text{approximation: at the end, one has 1009g!}} = 9g$

Atomic mass of Na $\approx 22.99 \frac{g}{mol}$

Atomic mass of Cl $\approx 35.45 \frac{g}{mol}$

Molar Mass of NaCl $\approx 22.99 \frac{g}{mol} + 35.45 \frac{g}{mol} = 58.44 \frac{g}{mol}$

$$\text{Moles} = \frac{9g}{58.44 \frac{g}{mol}} \approx 154\ mM$$

Concentration comparison

		mM	$\frac{g}{L}$
Sea water	NaCl	600 mM	3.5%
Fresh water	NaCl	<8.56 mM	<0.05%

↗ Similar to nerve cell

	Plasma		Intracellular		Molar Mass $\frac{g}{mol}$
	mM	$\frac{g}{L}$	mM	$\frac{g}{L}$	↳ electrons don't practically change the mass
Na^+	142 mM	3.26	10 mM	0.23	22.99
Cl^-	103 mM at nerve cell: 146	3.65	4 mM inside nerve cell: 14	0.14	35.45
K^+	4 mM	0.16	140 mM	5.474	39.10
HCO_3^-	24 mM	1.46	10 mM	0.61	61.02
Ca^{2+}	2.5 mM $\left(\hat{=} \frac{1}{2} \frac{meq}{L}\right)$	0.1	$0.5 \cdot 10^{-4}$ mM ↳ stored inside endoplasmic reticulum	0.002	40.08

Valency of 1: $\left(\hat{=} \frac{1\ meq}{L}\right)$ — Na^+, Cl^-, K^+, HCO_3^-

Valency of 2: — Ca^{2+} ↓ valency of 2

Polarization Relaxation in Liquids

Paul Debye Model

- polar molecules (dipoles) rotate in liquid with friction

⇒ relative permittivity becomes complex

frequency dependent! — limits at very low/high frequencies — energy storage (capacitors) — energy loss i.e. resistor loss

$$\underline{\varepsilon}_r(\omega) = \varepsilon_{r\infty} + \frac{\varepsilon_{r0} - \varepsilon_{r\infty}}{1 + j \cdot \omega \tau_\varepsilon} = \varepsilon_r'(\omega) - j \cdot \varepsilon_r''(\omega)$$

τ_ε ← polarization relaxiation time

$$= \varepsilon_{r\infty} + \frac{\varepsilon_{r0} - \varepsilon_{r\infty}}{1 + j\omega\tau_\varepsilon} \cdot \frac{1 - j\omega\tau_\varepsilon}{1 - j\omega\tau_\varepsilon}$$

$$= \varepsilon_{r\infty} + (\varepsilon_{r0} - \varepsilon_{r\infty}) \cdot \frac{1 - j\omega\tau_\varepsilon}{1 + \omega^2\tau_\varepsilon}$$

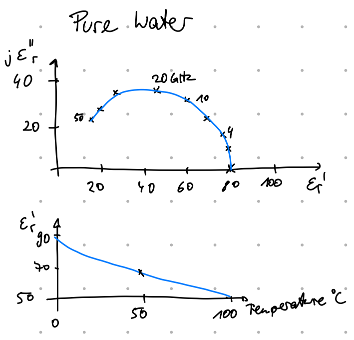
$$= \varepsilon_{r\infty} + \frac{\varepsilon_{r0} - \varepsilon_{r\infty}}{1 + \omega^2\tau_\varepsilon} - j\frac{(\varepsilon_{r0} - \varepsilon_{r\infty})\omega\tau_\varepsilon}{1 + \omega^2\tau_\varepsilon}$$

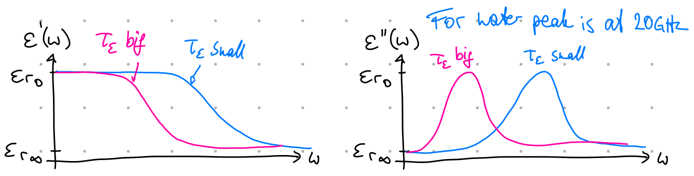
$$= \frac{\varepsilon_{r0} - \varepsilon_{r\infty} + \varepsilon_{r\infty}(1 + \omega^2\tau_\varepsilon)}{1 + \omega^2\tau_\varepsilon} - j\frac{(\varepsilon_{r0} - \varepsilon_{r\infty})\omega\tau_\varepsilon}{1 + \omega^2\tau_\varepsilon}$$

$$= \underbrace{\frac{\varepsilon_{r0} + \omega^2\tau_\varepsilon \cdot \varepsilon_{r\infty}}{1 + \omega^2\tau_\varepsilon}}_{\varepsilon'(\omega),\ \text{stored energy in capacitance}} - j\underbrace{\frac{(\varepsilon_{r0} - \varepsilon_{r\infty})\omega\tau_\varepsilon}{1 + \omega^2\tau_\varepsilon}}_{\varepsilon''(\omega),\ \text{loss}}$$

↳ 2x Zeros: $\omega^2 = -\frac{1}{\tau_\varepsilon} \cdot \underbrace{\frac{\varepsilon_{r0}}{\varepsilon_{r\infty}}}_{>1}$

2x Poles: $\omega^2 = -\frac{1}{\tau_\varepsilon}$

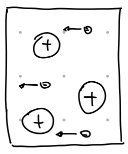




For water peak is at 20GHz

Conduction in Metals

Drude Model

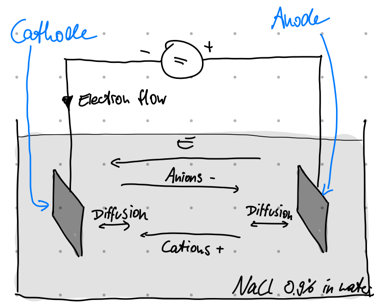


- Metals: immobile, positively charged ions, mobile, negatively charged electrons
- electrons like a gas moving freely
- electrons in random thermal motion, do not interact with each other
- electrons experience collisions with atoms after avg. time τ_{coll}, limiting response to electric field

$$\underline{\vec{J}} = \underline{\kappa}(\omega) \cdot \underline{\vec{E}}$$

$$\underline{\kappa}(\omega) = \frac{\overset{\text{electron density}}{n} \cdot e^2}{\underset{\text{electron mass}}{m}} \cdot \frac{\overset{\text{avg. time to collision} \approx 10^{-14}s}{\tau_{coll}}}{1 + j\omega\tau_{coll}}$$

Electrolytic experiment



NaCl : solute } Solution
Water : solvent }

Conduction in Electrolytes

- pure electrolytes: charge carriers are ions, not electrons
- ionic conductivity ∃ substantial transport of substances (especially at low freqs/DC)
- ions take part in complex chemical reactions when reaching metal → eg. electrolysis

	$\sigma\ [\frac{S}{m}]$
NaCl in water 25%	22
NaCl in water 0.9%	1.3
Blood 37%	0.7
Muscle 37%	0.4
Bone living	0.01
Tooth	0.005

Faraday's Law of Electrolytes

↳ relationship amount of altered substance at electrode during electrolysis and charge through electrolytic cell

$$\underset{\text{charge}}{Q} = \underset{\text{Faraday constant!}}{F} \cdot N$$ ← amount of substance in mol

$F \approx 96485 \frac{C}{mol}$ ← one mol of electrons has 96485C of charge!

$1\ mol \approx 6.022 \cdot 10^{23}$

Equivalent conductance

$$\Lambda_p = \frac{\kappa_p}{C_p} = \alpha_p \cdot |z_p| \cdot F \cdot u_p$$

e.g. $K^+ : \Lambda = 73.48 \cdot 10^{-4} \frac{m^2 S}{mol}$ @ 25°C

$Cl^- : \Lambda = 76.31 \cdot 10^{-4} \frac{m^2 S}{mol}$ @ 25°C

Ionic Conductivity

$$\kappa_p = \underset{\substack{\text{degree of}\\ \text{dissociation (0 to 1)}}}{\alpha_p} \cdot \underset{\substack{\text{valence}\\ \text{(oxidation number)}}}{|z_p|} \cdot \underset{\text{molar concentration}}{C_p} \cdot \underset{\substack{\text{Faraday}\\ \text{Constant}}}{F} \cdot \underset{\substack{\text{velocity achieved under}\\ \text{a unit electric field}}}{u_p}$$

p ← ion type p

Mobility $u_p\ [10^{-8} \frac{m^2}{Vs}]$ at 18°C

$H^+ \approx 33$, $Na^+ \approx 4.6$, $K^+ \approx 6.7$, $OH^- \approx 8.2$, $Cl^- \approx 6.8$

Example: 0.1 $\frac{mol}{L}$ NaCl in water (salty water)

- Concentration $C_p = 0.1 \frac{mol}{L} = 100 \frac{mol}{m^3}$
- Dissociation $\alpha = 1$ (NaCl is strong salt) ← From Gemini
- Valence : z_p : 1
- Mobility $Na^+ \approx 4.6$, $Cl^- \approx 6.8$ in $10^{-8} \frac{m^2}{Vs}$

$$\Rightarrow \kappa_{Na^+} \approx 1 \cdot 1 \cdot 100 \frac{\cancel{mol}}{m^{\cancel{3}}} \cdot 96485 \frac{\overset{As}{\cancel{C}}}{\cancel{mol}} \cdot 4.6 \cdot 10^{-8} \frac{\cancel{m^2}}{V\cancel{s}} = 0.4438 \frac{S}{m}$$

$$\Rightarrow \kappa_{Cl^-} \approx 1 \cdot 100 \frac{mol}{m^3} \cdot 96485 \frac{As}{mol} \cdot 6.8 \cdot 10^{-8} \frac{m^2}{Vs} = 0.656 \frac{S}{m}$$

$$\Rightarrow \kappa_{Total} = 0.4438 \frac{S}{m} + 0.656 \frac{S}{m} = 1.1 \frac{S}{m}$$

$$\vec{J}_{el.\ cond} = \vec{J}_{drift} = \kappa_{ionic} \cdot \vec{E}$$

Nernst-Planck Equation

↳ flow of ions in electrolytic solution is due to
1. concentration gradient (leading to diffusion current)
2. presence of electric field (leading to drift current)

$$\vec{j}_p^{\,total} = \vec{j}_p^{\,diff} + \vec{j}_p^{\,drift}$$

$$\vec{j}_p^{\,diff} = -D_p \cdot z_p \cdot F \cdot \nabla C = -\frac{z_p}{|z_p|} \cdot RT \cdot u_p \cdot \nabla C_p$$

$$\vec{j}_p^{\,drift} = \underbrace{\alpha_p \cdot |z_p| \cdot C_p \cdot F \cdot u_p}_{\kappa_p} \cdot \underbrace{\nabla\phi}_{\vec{E}}$$

Nernst Equation & Nernst Potential

↗ calculates Nernst potential

↓ magnitude of equilibrium potential difference

$$\Delta\phi = \phi_1 - \phi_2 = \pm \frac{RT}{F} \cdot \ln\left(\frac{c_1}{c_2}\right) = \pm \frac{k_B T}{e} \cdot \ln\left(\frac{c_1}{c_2}\right)$$

− for cations, + for anions

↳ different Nernst equations for each specific ion

Eg. Na^+ (Intracellular: 10mM, Extracellular: 142mM)

$$\rightarrow \Delta\phi_{Na^+} = -\frac{8.314 \frac{J}{\cancel{K mol}} \cdot \overset{37°C}{310 \cancel{K}}}{96485 \frac{C}{\cancel{mol}}} \cdot \ln\left(\frac{10mM}{142mM}\right) \approx 70.874mV$$

$$\Delta Q_{K^+} = \ldots \cdot \ln\left(\frac{140mM}{4mM}\right) \approx -94.97mV$$

Diffusion current

↳ random thermal motions of ions lead to diffusion currents if there are local variations in concentration

$$\vec{j}_p^{\,diff} = -\underset{\text{Diffusion Coefficient}}{D_p} \cdot z_p \cdot F \cdot \nabla C$$

$$D_p = \frac{u_p}{|z_p|} \cdot \frac{\overset{\text{ideal gas const., absolute temperature}}{RT}}{F} = \frac{u_p}{|z_p|} \cdot \frac{\overset{\text{Boltzman const.}}{k_B} T}{\underset{\text{elementary charge}}{e}}$$ ← Einstein Relation

Ion	$D\ [\frac{cm^2}{sec}]$ @25°C
Na^+	$1.33 \cdot 10^{-5}$
K^+	$1.96 \cdot 10^{-5}$
Cl^-	$2.03 \cdot 10^{-5}$
KCl	$2.03 \cdot 10^{-5}$ } $0.002 \frac{mole}{L}$
NaCl	$1.58 \cdot 10^{-5}$ }

Electrochemical potential

↳ rate of change of free energy of system with respect to change in number of its elements

Equilibrium conditions ($\vec{j}_p^{\,total} = 0$)

$$\Rightarrow z_p \cdot F \cdot \phi + RT \cdot \ln(C_p) = \underset{\text{Electrochemical potential}}{\underline{ECP}} = const.$$

⇒ measured in $\frac{J}{mol}$

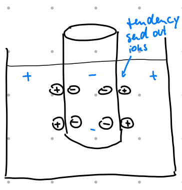
Equilibrium Potentials

Metal/Ion	Volt
Li/Li^+	-3.05
K/K^+	-2.93
Na/Na^+	-2.71
H_2/H^+	0

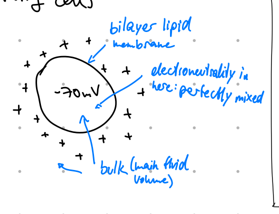
Electric Double Layers (EDL)

↳ layer of ions/electrons facing oppositely charged ion layers
Counter ions

e.g. Metal/Ion Half Cell

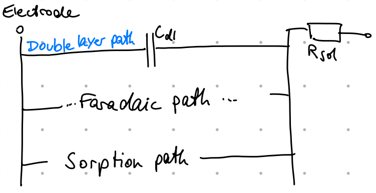


e.g. living cells

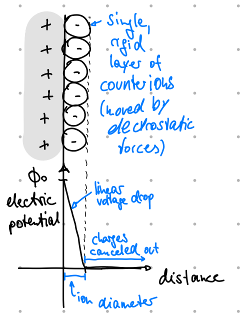


Electrode Equivalent Circuit

Electrode wire → solution

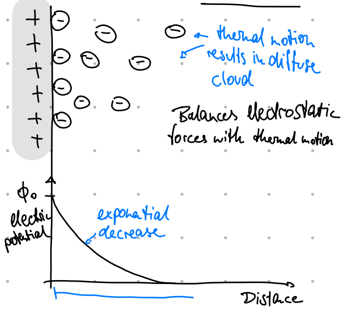


Helmholtz Model



Flaws:
- ignores thermal motion
  ↳ only electrostatic attraction
- predicts constant capacitance regardless of voltage & concentration

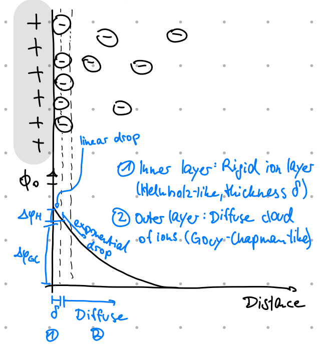
Gouy-Chapman's Model



Flaws:
- predicts impossibly high ion densities for highly charged DLs
  ↳ capacitance unphysically rises exponentially
- doesn't take ion size into account

Stern's Model

↳ Hybrid approach, used nowadays



① Inner layer: Rigid ion layer (Helmholz-like, thickness δ)
② Outer layer: Diffuse cloud of ions (Gouy-Chapman-like)

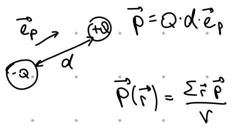
→ Prevents infinite concentration

Electric dipoles, polarization

↳ can be visualized as two charges of opposite polarity separated by tiny distance

- Electric dipole moment p gives direction, magnitude
- Electric polarization P is vector field: local density of electric dipole moments in material

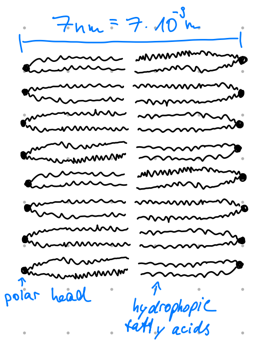
$$\vec{D} = \varepsilon_0 \cdot \vec{E} + \vec{P} \qquad \vec{P} = \varepsilon_0 (\varepsilon_r - 1) \cdot \vec{E}$$



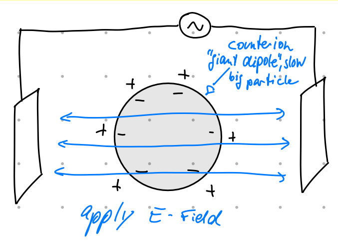
$$\vec{p} = Q \cdot d \cdot \vec{e}_p$$

$$\vec{P}(\vec{r}) = \frac{\Sigma_{\vec{r}} \vec{p}}{V}$$

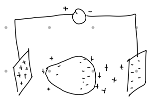
Bilayer Lipid Membrane (BLM)



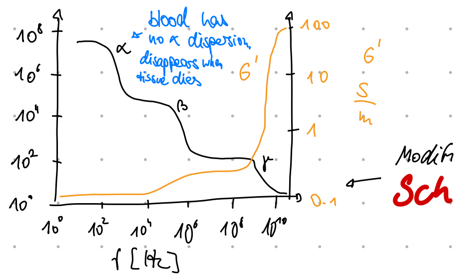
Electric dispersion in tissues



Counterion effect
↳ Zelle verformt sich, da sich Membran makroskopisch lädt und bewegt sich evtl.

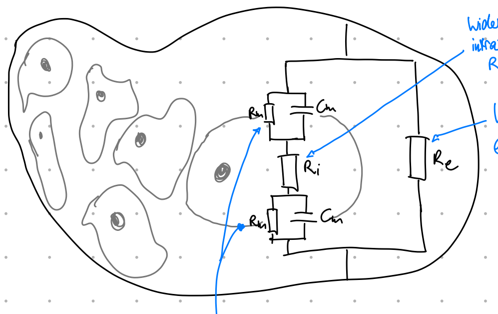


Type	Frequency	Mechanism
α	mHz - kHz	•Counterion effects •active cell membrane effects & gated channels, intracellular structures ionic diffusion, dielectric losses
β	kHz - 100MHz	Maxwell-Wagner effects: passive cell membrane capacitance, intracellular organelle membranes
γ	0,1 - 100 GHz	Dipolar mechanisms in polar media such as water, salts, proteins



Modified from **Schwan**

Equivalent circuits of tissues



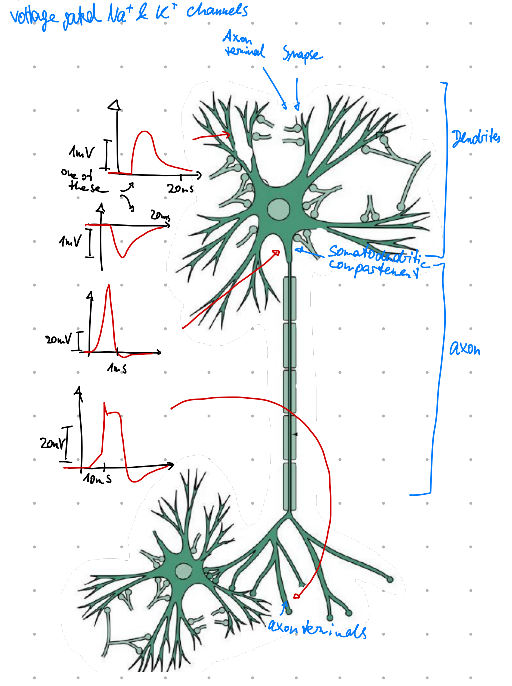
Widerstand des intrazellulären Raums

Widerstand des extrazellulären Raums

Transmembran-Widerstand bzw. Kapazität

Signal Transmission in Neurons

- motor nerve axon has voltage gated Na^+ & K^+ channels



Axon Signal Velocities

	e.g.	Diameter (μm)	Velocity ($\frac{m}{s}$)
Myelinated	Skeletal muscle	15 - 20	60 - 100
	Vibration, high precision touch	5 - 15	30 - 90
	Deep pressure	1 - 5	6 - 30
Unmyelinated	Sympathetic, pain, tickle, pressure	0.5 - 2	0.5 - 2

↳ Myelin: lipid-rich substance around axon for better electrical insolution

Elektrische Dipole

statisches Dipolfeld verringert sich mit $\sim \frac{1}{r^3}$ ← Entfernung

Elektrisches Dipolmoment $\vec{P}$ mit Betrag & Richtung

$$\vec{P} = q \cdot \vec{d}$$

Q(-) $\vec{d}$ Q(+) Negativer zu Positiver Ladung

Für $|r| \gg d$: Feld nur noch von $\vec{p}$ abhängig (nähert sich Punktdipol)

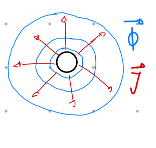
Stationary Current (field)

↳ due to time-independent boundary conditions given by CURRENT SOURCES or impressed ELECTRIC POTENTIALS (voltage sources)

$$\nabla \cdot \vec{J} = 0$$ ← in source-free regions

$$\nabla \cdot \vec{J} = I_V(\vec{r})$$ isotropic volume current source

$$\vec{J}(\vec{r}) = \frac{I_{0,\vec{r}}}{4\pi} \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

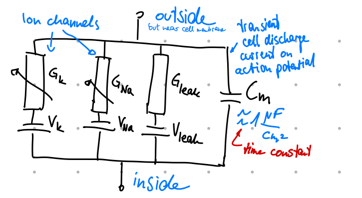


$$\Phi(\vec{r}) = \frac{I_{0,\vec{r}}}{4\pi\kappa} \frac{1}{|\vec{r} - \vec{r}'|}$$

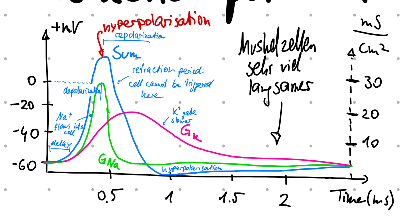
Hudgkin & Huxley: Cell membrane model

↳ measured voltage inside nerve cell for the first time

- discovered ion flux phenomenon



Cell action potential



Cell Potential

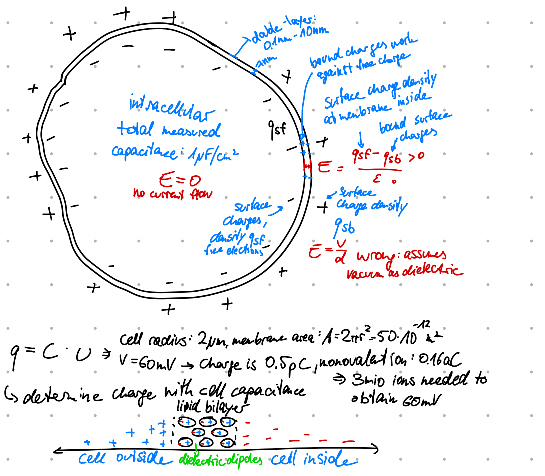
$$V = -61 \cdot \log_{10}\left(\frac{C_e}{C_i}\right) [mV]$$

C_e: extracellular ion concentration; C_i: intracellular ion concentration

If more than one ion species: permeability ($\frac{m}{s}$)

$$V = -61 \cdot \log_{10}\left(\frac{C_{iNa^+} \cdot P_{Na^+} + C_{iK^+} \cdot P_{K^+} + C_{iCl^-} \cdot P_{Cl^-}}{C_{eNa^+} \cdot P_{Na^+} + C_{eK^+} \cdot P_{K^+} + C_{eCl^-} \cdot P_{Cl^-}}\right)$$

Zellpotential von erregbaren Zellen -40mV bis -90mV (durch Präsenz von Makromolekülen, Donnan-Effekt)



$q = C \cdot U \Rightarrow$ cell radius: 2μm, membrane area: $A = 2\pi r^2 = 50 \cdot 10^{-12} m^2$
V = 60mV → charge is 0.5pC, monovalent ion: 0.16aC
⇒ 3Mio ions needed to obtain 60mV
↳ determine charge with cell capacitance

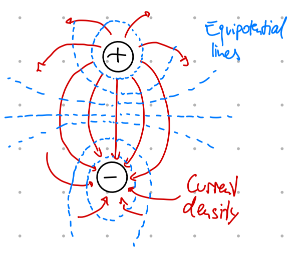
Dipole Source

$$\Phi(\vec{r}) = \frac{\vec{P}_{1,r}}{4\pi\kappa} \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{P}_1 = I_0 \cdot d \cdot \vec{e}_1 \qquad d \ll |\vec{r} - \vec{r}'|$$

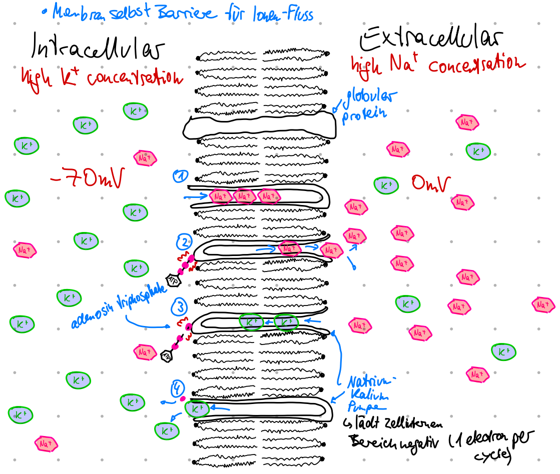
$-I_0$ $\vec{e}_1$ $+I_0$, distance d

$$\vec{J}(\vec{r}) = \frac{1}{4\pi}\left(\frac{3((\vec{r}-\vec{r}')\cdot\vec{P}_{1,r})\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^5} - \frac{\vec{P}_{1,r}}{|\vec{r}-\vec{r}'|^3}\right)$$



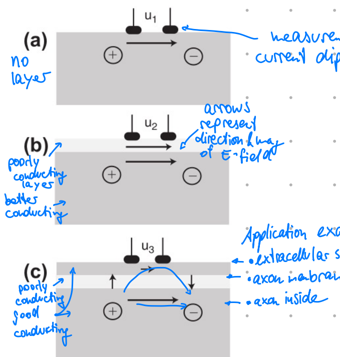
BLM Channels

- Kalium-Natrium-Pumpe (70% ATP-Verbrauch) ← Millionen in einer Zellmembran
  ↳ hält negatives Potential in Zelle
- Wasser-Kanäle
- Membran selbst Barriere für Ionen-Fluss



Current dipole signal transfer

(not in lecture)

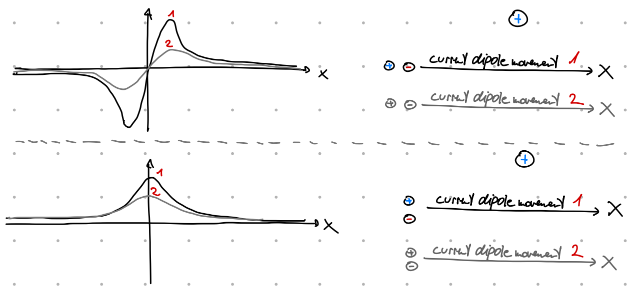


Application example:
- extracellular space
- axon membrane
- axon inside

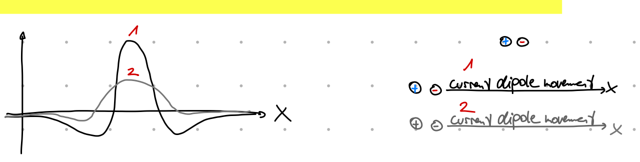
Ion Channel

- filter + gate, highly selective: permeability for K^+ 1000-10000 higher than for Na^+
- K^+-channel: 200Mio ions/sec)
  ↳ if open, 1ms: 0.2Mio ions
- monovalent ion: 0.16aC (attocoulomb $= 10^{-18}$C)
  ↳ charge of 32fC
- If one cell has 10Mio channels: charge of 0.32μC moves intracellular to extracellular
- gate normally closed, but can be opened with voltage or ligand
- motor nerve axon has voltage gated Na^+, K^+ channels

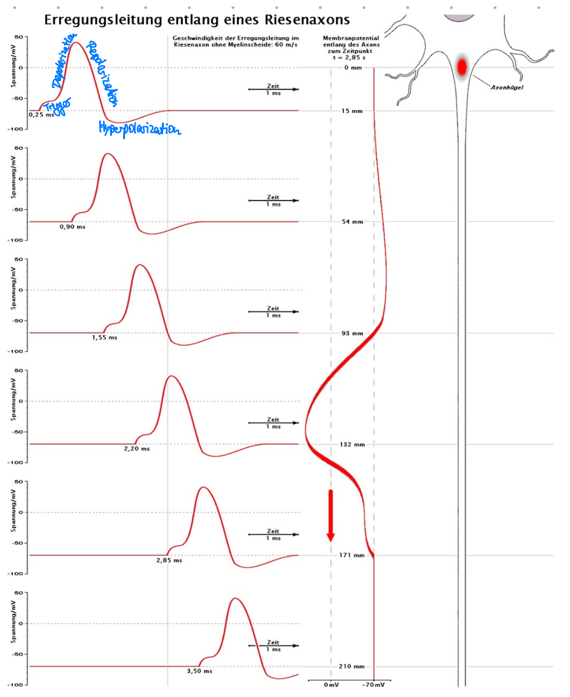
Unipolar Potentials of current dipoles



Bipolar potentials of current dipole



Erregungsleitung Riesenaxon

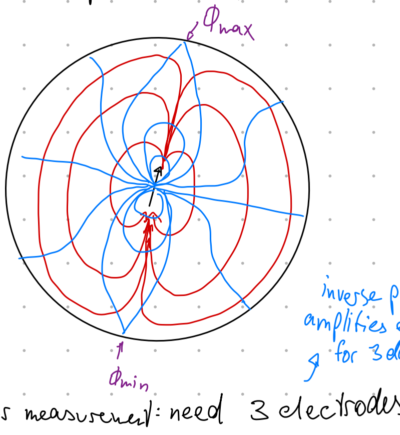


Reobase: Threshold, um Zell-Aktionspotential auszulösen wenn Puls unendlich lang

Chronaxie: Doppelte Reobase

Current dipole in conducting sphere

Current dipole in [Am]



inverse problem amplifies error for 3 electrodes

For measurement: need 3 electrodes,
↳ perpendicular are best (more equipotential lines)

↱ Communication: Both nerves and hormones

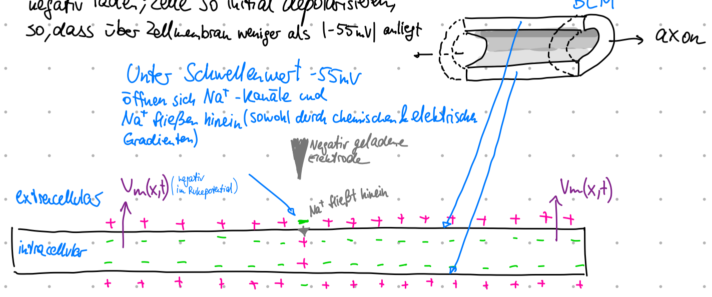
Cell Action Potential & Propagation

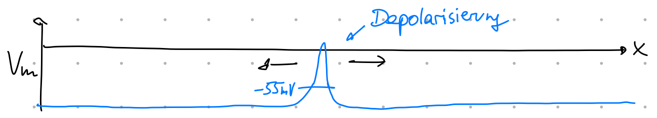
- hormones: slow, broadcasting
- nerves: quick, peer to peer
- not excitable cells: adipose, connective tissue, blood
- excitable, polarized cells: nerve, muscle, gland cells

1. Trigger voltage opens Na^+-Channels: Na^+ "voltage source is active"
2. Depolarization: Na^+ quickly flows inwards following Nernst Potential of +60mV
   ↳ Few ions need to flow since only membran potential needs to change
3. Trigger voltage activates K^+-Channels delayed
4. Na^+ channels close before reaching Na^+-Nernstpotential at 60mV
   ↳ only goes to ≈+30mV
4. Repolarization: K^+ flows outwards following -90mV Nernst-Potential
5. Na-K-Pumps slowly recharges cell, pumps Na^+ & K^+ ions back against gradient

Triggering an action potential

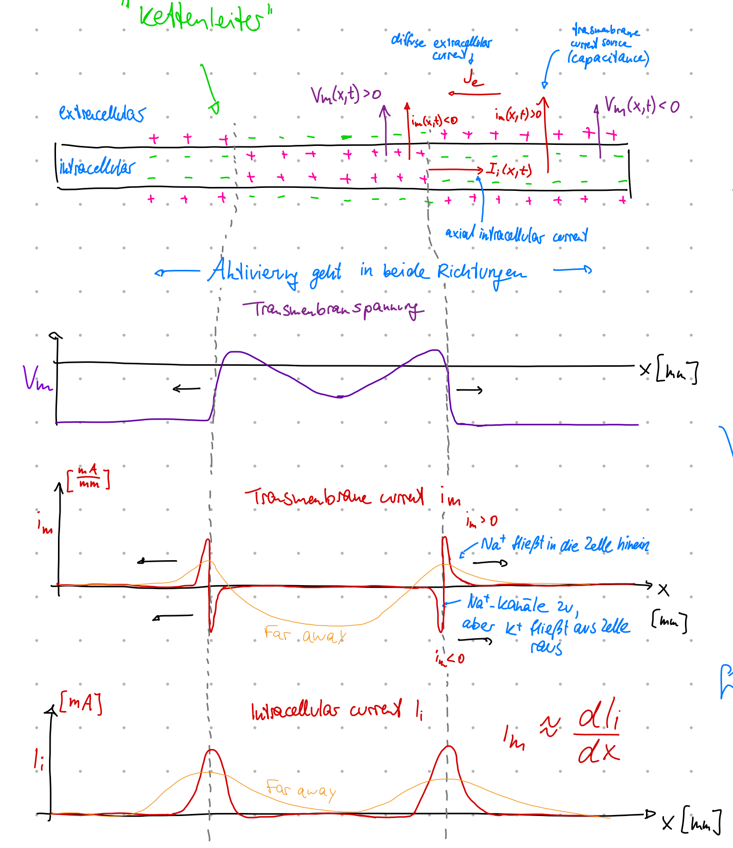
1. Mit negativ geladener Elektrode extrazelluläre Flüssigkeit negativ laden; Zelle so initial depolarisieren, so, dass über Zellmembran weniger als |-55mV| anliegt



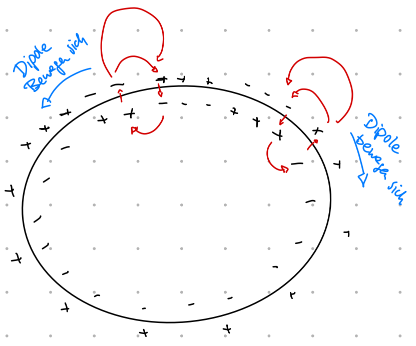


2. Aktivierte Na^+-Kanäle sorgen für Anstieg des Potentials in Zelle auf 30mV

"Kettenleiter"



$$I_m \approx \frac{dI_i}{dx}$$



$$V_m(x) = \frac{\Phi_i(x)}{\Phi_e(x)}, \quad I_i(x) = -\frac{1}{r_i} \cdot \frac{dV_m(x)}{dx} [A]$$

$$i_m(x) = -\frac{dI_i}{dx}(x) = \frac{1}{R_i} \cdot \frac{d^2V_m(x)}{dx^2}$$

$$\Rightarrow \frac{d^2V_m(x,t)}{dx^2} - r_i C_{km} \frac{dV_m(x,t)}{dt} - g\,V_m(x,t) = 0$$

Extracellular potential of fibers

↳ cylindrical fiber in unbounded, homogeneous, isotropic medium

$$\Phi_e(\vec{r}) = \frac{1}{4\pi\kappa_e} \cdot \int_L \frac{i_m(\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r}'$$

transmembrane current

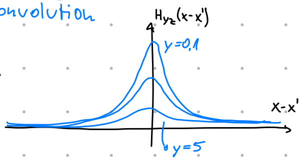
conductivity of medium

If fiber is on x-Axis:

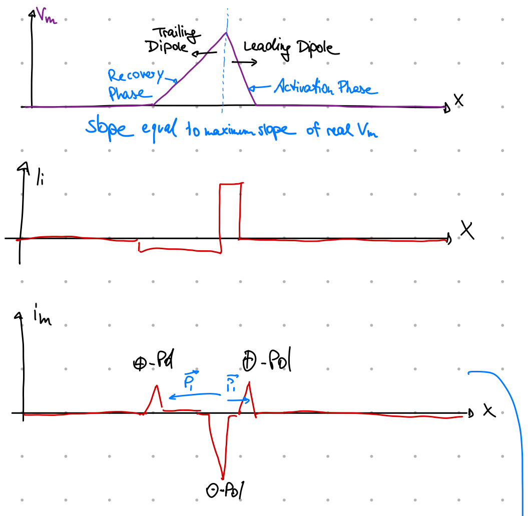
$$\Phi_e(x,y,z) = \frac{1}{4\pi\kappa_e} \int_L \frac{i_m(x')}{\sqrt{(x-x')^2+y^2+z^2}} dx'$$

$$= \frac{1}{4\pi\kappa_e} \cdot \int_L \underbrace{H_{yz}(x-x') \cdot i_m(x')}_{\text{convolution}} dx'$$

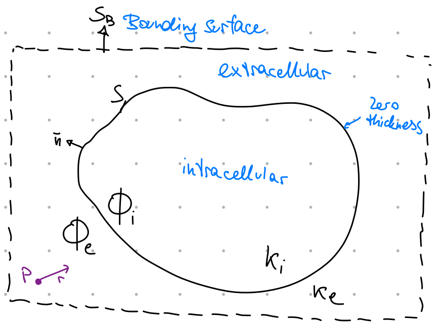
$$H_{yz}(x-x') = \frac{1}{\sqrt{(x-x')^2+y^2+z^2}}$$



Triangularized Action Potential



Excitable Cell Model



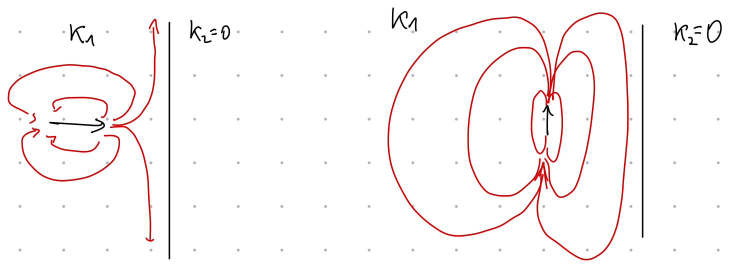
STROMDIPOL

Kombination entgegengesetzt gleicher punktförmiges Stromquellen in einem endlichen Abstand

Stromquelle → Stromsenke

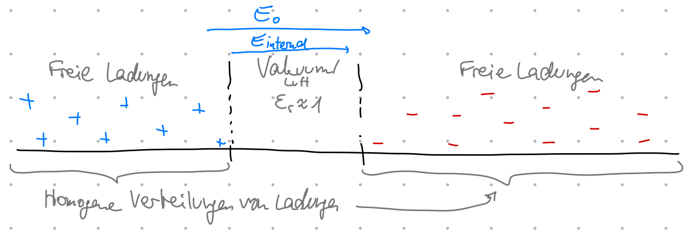
Current dipole near boundaries

Langenscheidt



Dielektrikum im Kondensator verändern

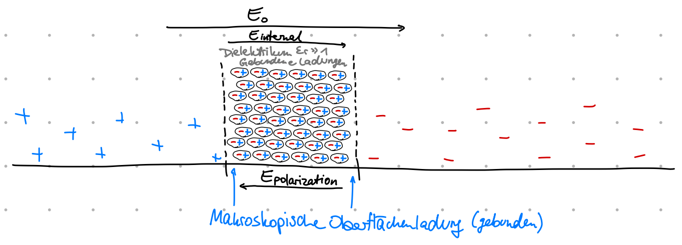
1. a. Mit Spannungsquelle konstante Ladung auf Elektroden befördern
   b. Spannungsquelle entfernen



- $E = U \cdot d \quad C = \frac{Q}{U} = \overset{\varepsilon_0 \cdot \varepsilon_r}{\varepsilon} \cdot \frac{A}{d}$
- Ladung Q bleibt konstant!

$\Rightarrow E_{internal} = E_0$

2. Dielektrikum mit $\varepsilon_r > 1$ einbringen



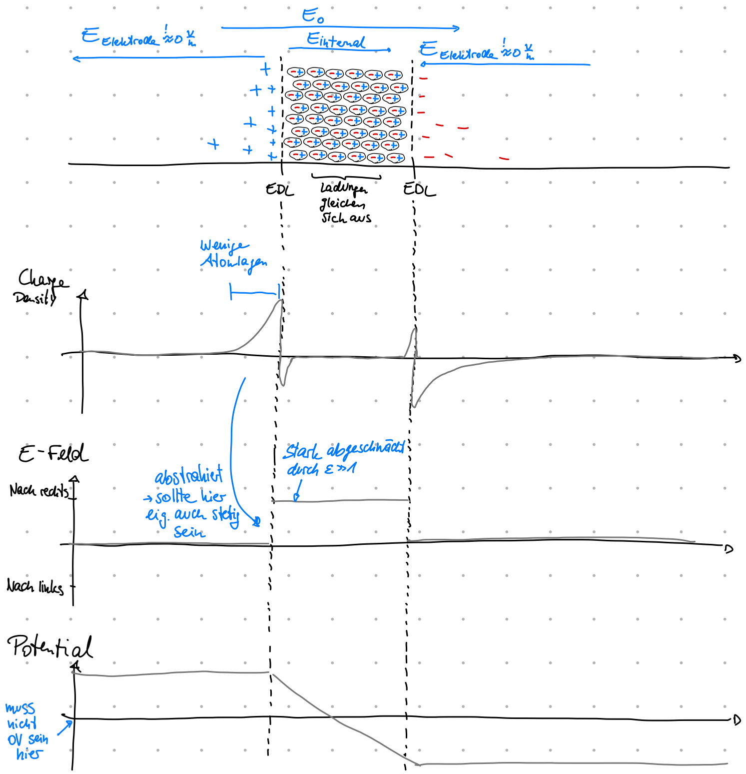
Stärke des Dipolmoments

$$E_{internal} = E_0 - E_{Polarization} = \frac{E_0}{\varepsilon} \qquad E_{pol} = E_0 - \frac{E_0}{\varepsilon}$$

$E_{internal} < E_0$!

$$D = \varepsilon E + P \qquad = \varepsilon \frac{U}{d} = \frac{Q}{A}$$

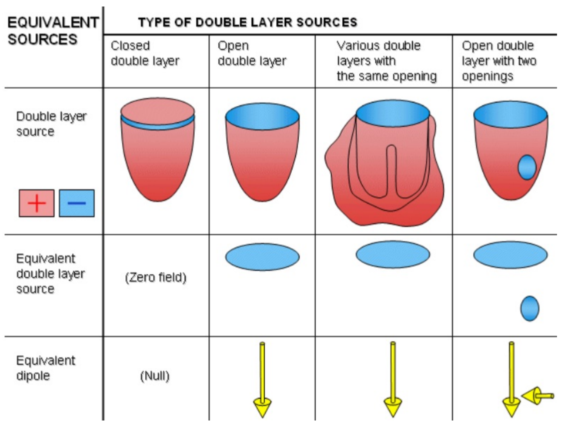
3. Ladungen in den Elektroden verteilen sich um



$$\varphi(x) = \int E(x)\,dx + C \qquad \frac{dE(x)}{dx} \cdot \varepsilon = \rho(x) \qquad \rho_{line}(x) = \varepsilon \int E(x)\,dx$$

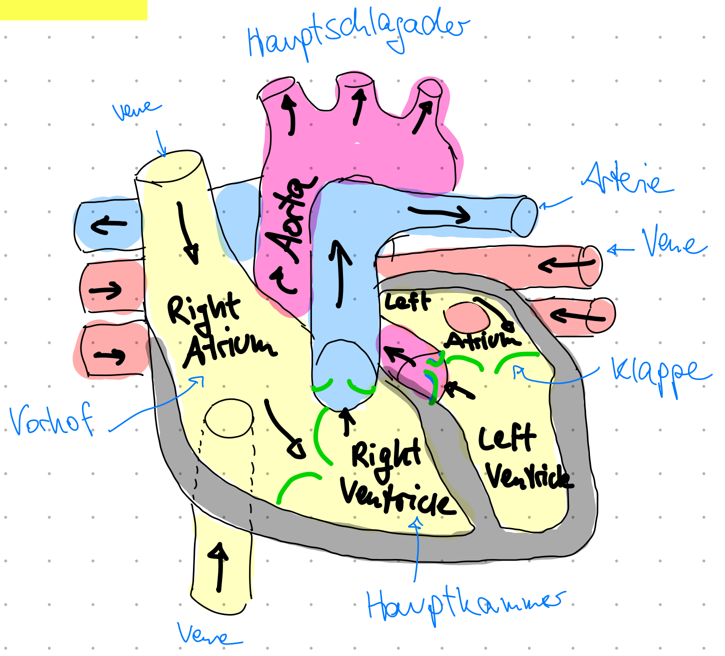
$$\rightarrow \varphi(x) = \frac{\rho_{line}(x)}{\varepsilon} + c$$

Double layer sources

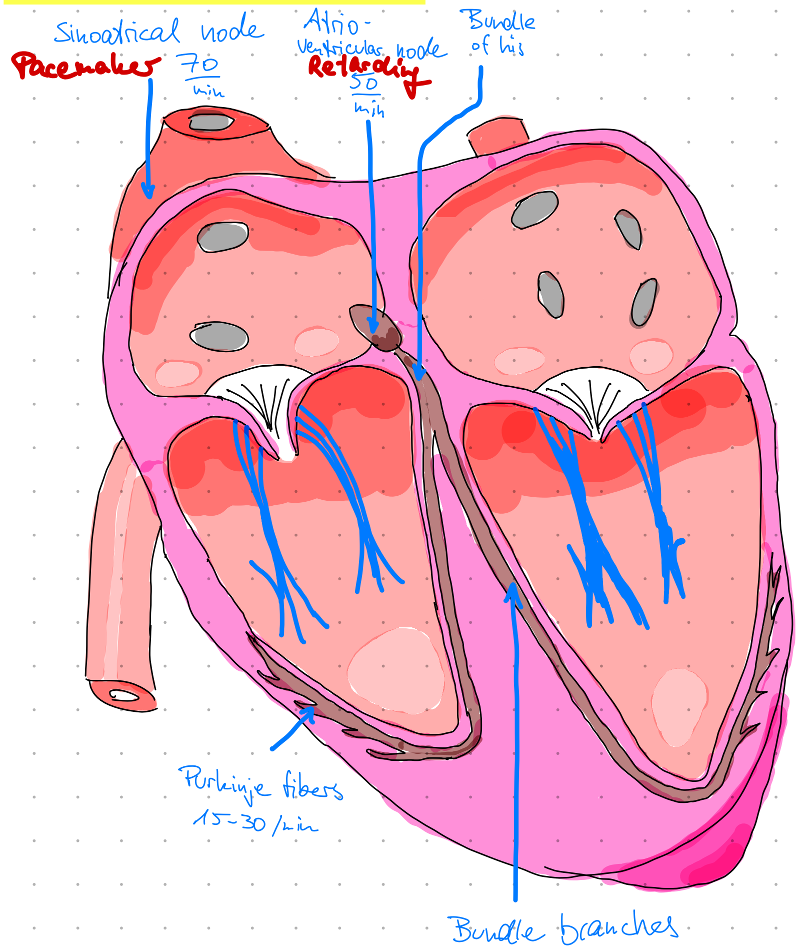


EQUIVALENT SOURCES	TYPE OF DOUBLE LAYER SOURCES			
	Closed double layer	Open double layer	Various double layers with the same opening	Open double layer with two openings
Double layer source 🟥 + 🟦 −				
Equivalent double layer source	(Zero field)			
Equivalent dipole	(Null)			

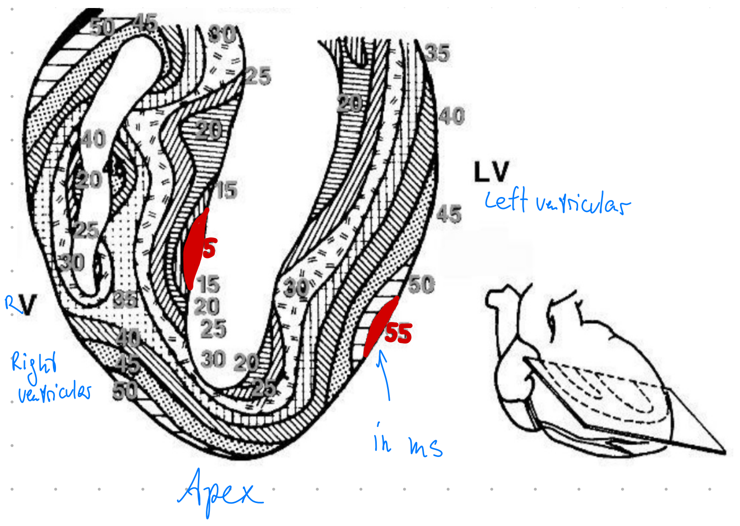
The heart



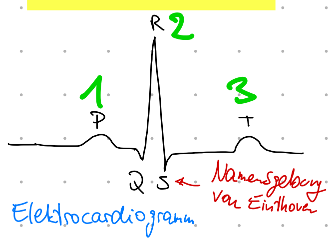
Heart conduction system



Isochronous Lines of Heart Activation

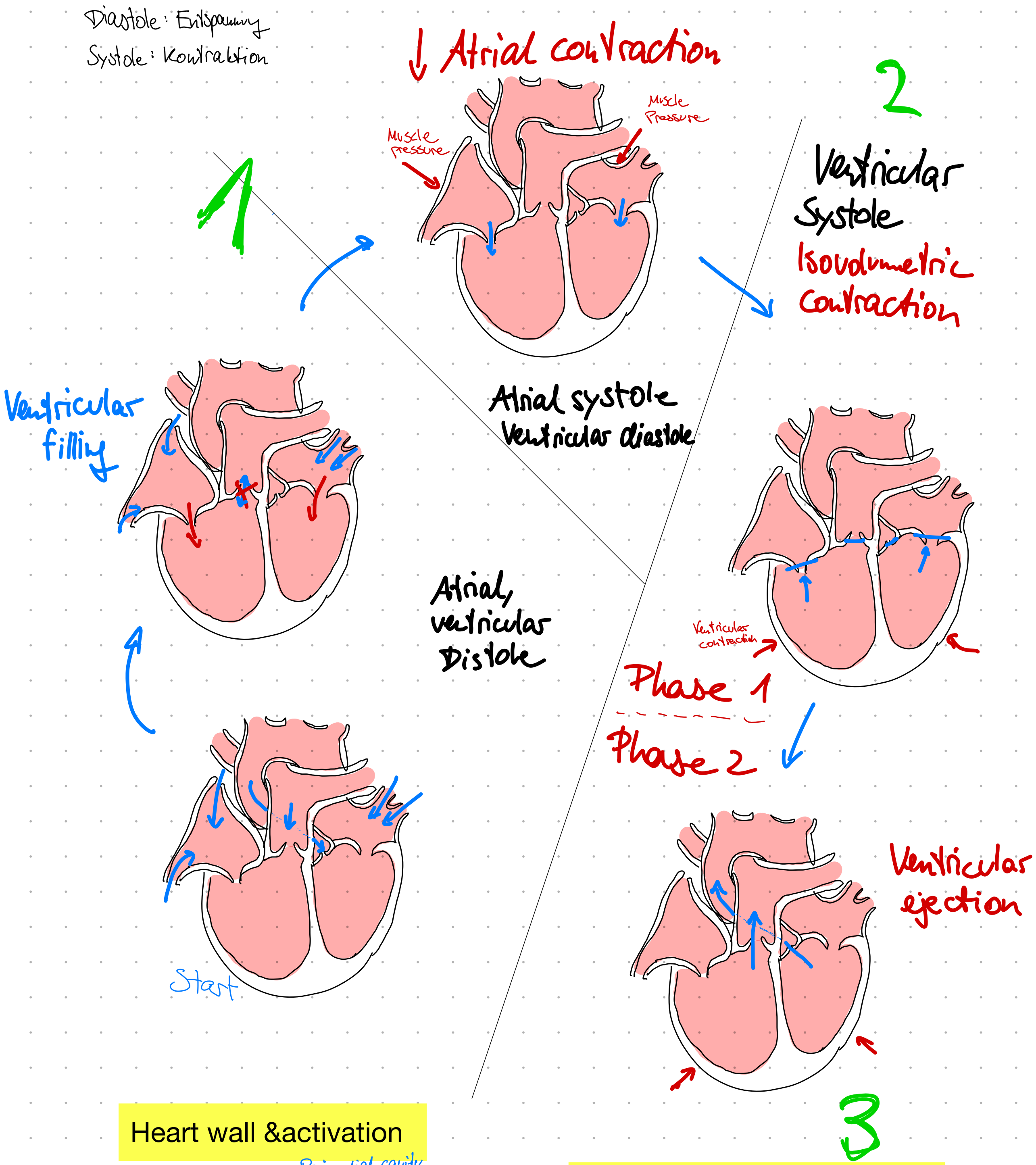


Cardiac Cycle

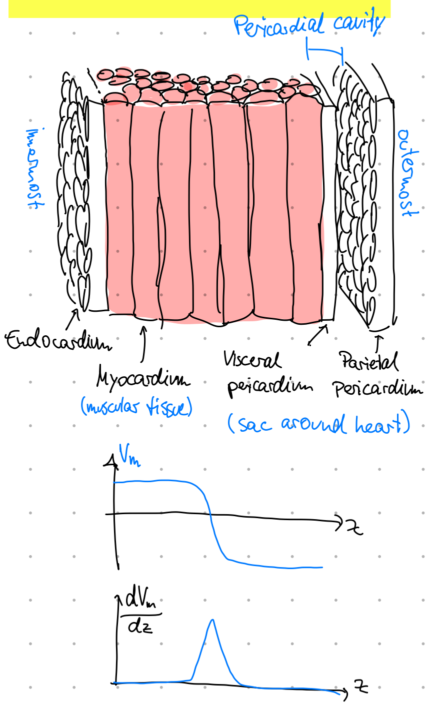


Diastole: Entspannung
Systole: Kontraktion

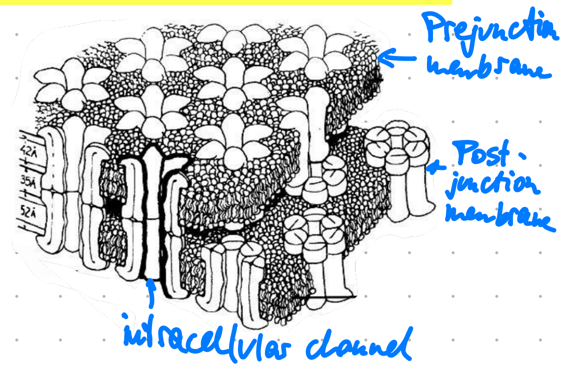
1. P : Atrial contraction
2. Zwischen PQ: Verzögerung zur Vermeidung simultaner Kontraktion
3. Q,R,S: Ventricular contraction
   ↳ Schnelle Ausbreitung der Erregung in His-Bündel & Purkinje-Fasern
4. T : Ventricular ejaculation

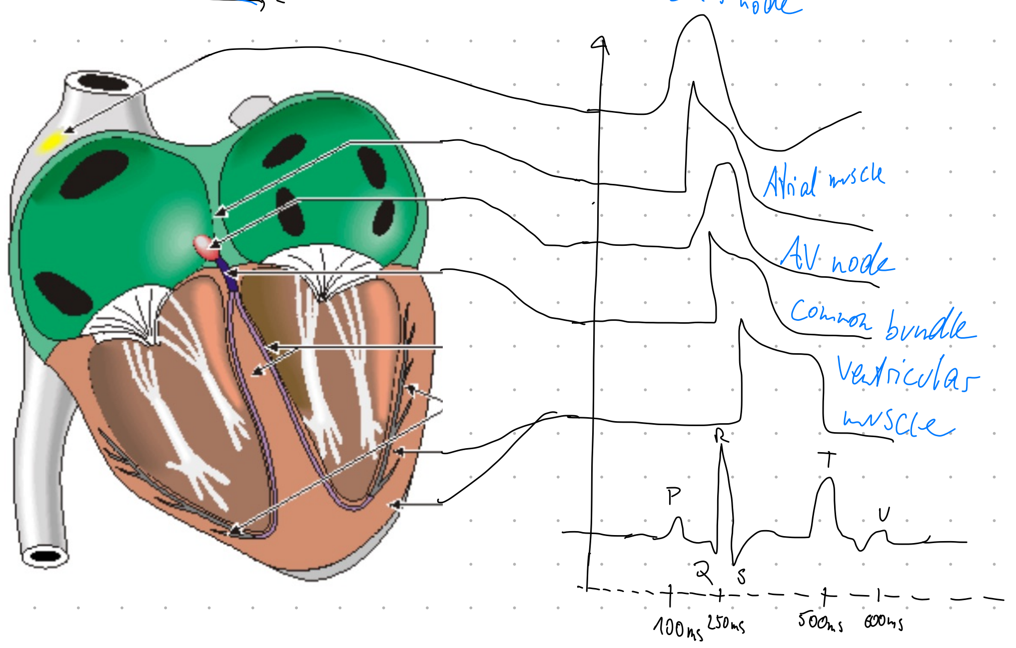


Heart wall &activation

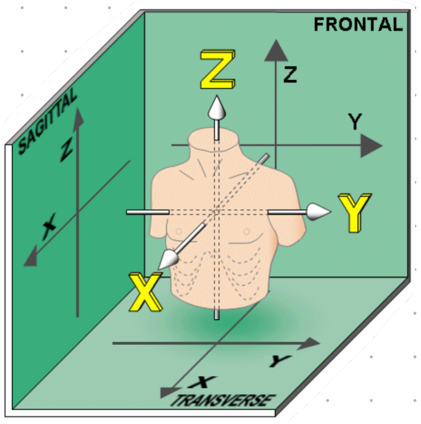


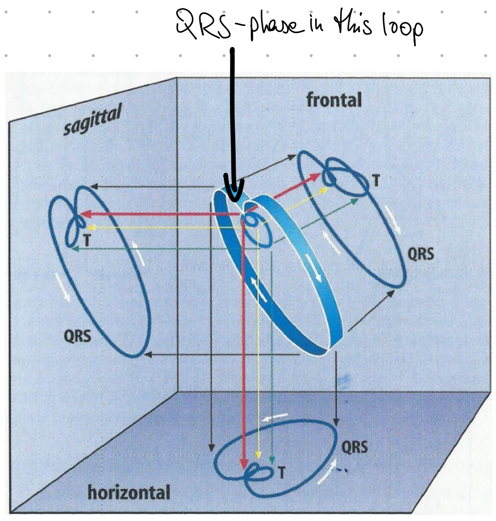
Intracellular cardiac junction





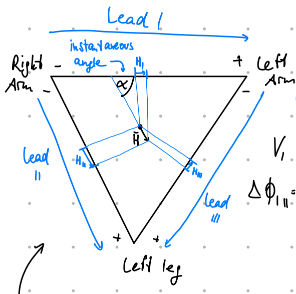
Heart vector





Einthoven's triangle

↳ solution to inverse problem



$V_I = \bar{H}_I \cdot \bar{L}_I$

$\Delta\phi_{I\,II} = \Phi_I - \Phi_{II} = \vec{p}_1 \cdot \vec{L}_{I\,II}$

QRS of signal used (closed loop in frontal plane)

Lead vector (fields)

lead voltage, lead vector $[\frac{\Omega}{m}]$, in Am

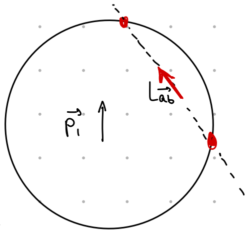
$$\nabla\Phi_{ab} = \Phi_a - \Phi_b = \vec{p}_1 \cdot \vec{L}_{ab}$$

Lead Vector: Gradient of Electric potential defined by reciprocally inserting/extracting a unit current I_0 using lead a-b

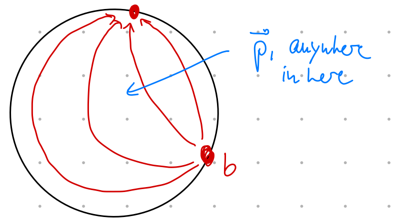
Reciprocal potential: Lead field

↳ Gradient: Lead vector field

$$\vec{L}_{ab}(\vec{r}) = -\frac{\vec{J}_{reci}(\vec{r})}{\kappa \cdot I_0}$$

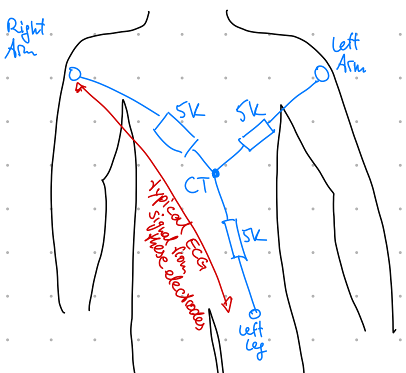


↳ Simple, homogeneous material

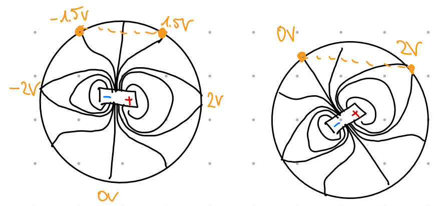


$$\Delta\Phi_{ab}(\vec{r}_{p_1}) = \Phi_a - \Phi_b = \vec{p}_1 \cdot \vec{L}_{ab}(\vec{r}_{p_1})$$

Electrocardiography



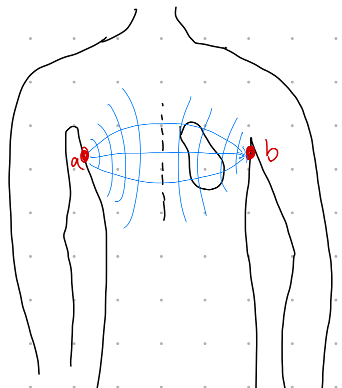
Spherical model for ECG leads



$\Delta\Phi_{ab} = \Phi_a - \Phi_b$

Einthoventriangle

Lead vector field in human



Lead field (lead I):

∿ - current flow field, introduced at b

Further clinical applications

- ECG (electro cardiography)
- EEG (electroencephalography)
- EIT (electrical impedance tomography)
- impedance cardiography
- ...

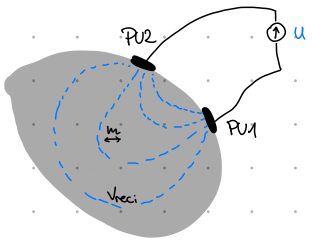
Sensitivity (field)

↳ normalized with respect to I_0

↳ given: current dipole & pair of leads

sensitivity field

$$\vec{S}_{ab}(\vec{r}_{\vec{p}_1}) = \kappa \cdot \vec{p}_1 \cdot \vec{L}_{ab}(\vec{r}_{\vec{p}_1}) \cdot \frac{1}{I_0} = -\vec{p}_1 \cdot \vec{J}_{reci}(\vec{r}_{\vec{p}_1}) \cdot \frac{1}{I_0^2}$$



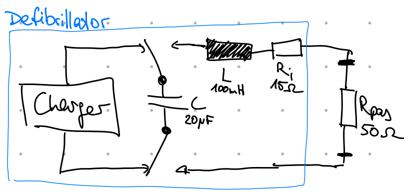
Electrical therapies

Cardiac pacemaking

- typical current amplitudes: 5mA
- lead-tips in right atrium / right ventricle of heart

Cardiac Defibrillation

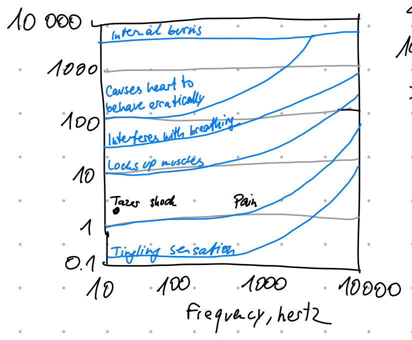
- > 50A peak possible



Electrosurgery

- high-freq current to cut, dry, coagulate, etc
- 20-80W, 0.5-5MHz, <400mA

Electroshock weapons (Tazer)



<100kHz : Elektrostimulation

100kHz-5MHz : Erhitzung des gesamten Körpers

>5MHz : Erhitzung der Oberflächen

- Rheobase & Chronaxie pulses get shorter when more current
- E-fields <100kHz: stimulation of cells
  tissue heating/burns, loss of muscle control, nerve damage
- most important vital organs sensitive! Heart, lung, brain stem
- >100kHz : tissue heating dominant

Standards, regulations

- ICNIRP: International commission on non-ionizing radiation protection
- IEEE: Institute of Electrical and Electronics Engineers
- Bundesgesetzblatt:
  Bundes-Immissionsschutzgesetz)
- ANSI: American National Standards Institute

Current limits: Both feet: 2A rms @ 3-100kHz,
0.2A rms @ 0.1-100MHz
(controlled environment)

50V, 15mA: Muscles let go

250V, 50mA: Macroshock at heart, lung
brain stem

Site dependence of skin impedance
(at 10Hz)

Typische Permeabilitätswerte?

Polarisierung Plane wave SAR strength Prop.

Rheobase/Chronaxie?

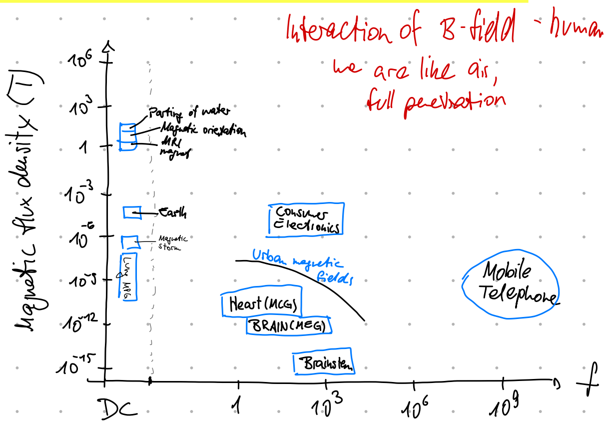
Near/Farfield

Susceptibility

magnet
magnitude
magnification

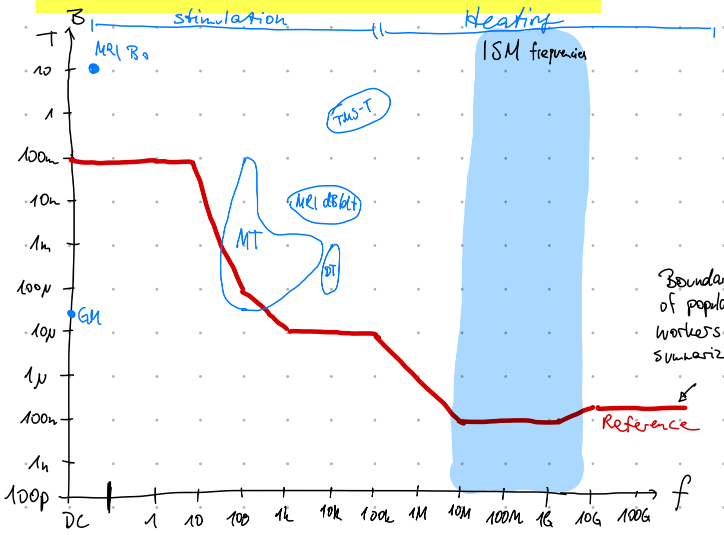
Biomagnetic fields & applications

Interaction of B-field - human:
we are like air,
full penetration



MPG: Magneto pneumography MCG: Magnetocardiography
MEG: Magnetoencephalography

Magnetic fields in medical devices



- GM: Geomagnetic field (of earth)
- MT: Magnet therapy
- MRI: Magnetic resonance imaging
- TMS-T = Transcranial therapeutic magnetic stimulation
- Magnetic device tracking

Medical uses of magnetic fields

- static magnetic fields B_0
- ELF magnetic field B_{eff}
- Magnetic transients $\frac{dB}{dt}$
- RF
- Microwave (µW)

- diathermy
- magnetic navigation
- device tracking
- capsular endoscope
- MT, TMS
- tracking nanoparticles

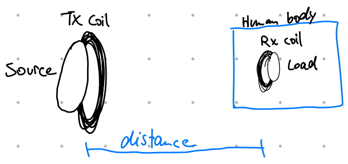
Magnetic field sensing with coils

- Near-field probes
- H-Field Probe for MRI
- inductive pickup coils
- Hall effect sensors
- Fluxgate sensors
- magnetoresistance sensors
- ...

Wireless Power Transfer with Coils

(WPT)

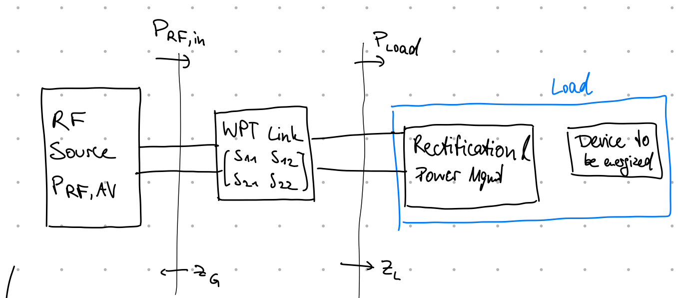
example capsule endoscopy



Efficiency:

$$\eta = \frac{P_{Load}}{P_{RF,in}} = |S_{21}|^2$$

Transmission Scattering Parameter



↳ Nicht nur Power, sondern auch Daten übertragen!

Biological effects of magnetic fields

Static homogeneous magnetic fields

→ Torque (Magnetic orientation of biological cells)

$$T = -\frac{1}{2\mu_0} B^2 \Delta x \cdot \sin(2\theta)$$

Static inhomogeneous magnetic fields

→ Force (Parting of water)

$$F = \frac{x}{\mu_0} \cdot (\text{grad } B) \cdot B$$

Time-Varying Magnetic field

→ Eddy currents (nerve stimulation)

$$J = -6 \cdot \frac{B}{t}$$

→ Heat $SAR = 6 \cdot \frac{E^2}{\rho}$

Multiplication of magnetic fields and other energy

- Photochemical reactions with radical pairs
- Singlet-triplet intersystem crossing (yield effect of cage product & escape product)

Biot-Savart Law

↳ calculate magnetic fields of DC/LF currents

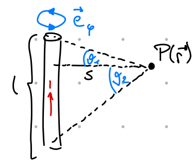
$$\vec{B}(\vec{r}) = \frac{\mu_0 \mu_r}{4\pi} \iiint_{\infty} \frac{\vec{J}(\vec{r}\,')dV' \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

For line currents:

$$\vec{B}(\vec{r}) = \frac{\mu_0 \mu_r}{4\pi} \int_C \frac{I(\vec{r}\,')d\vec{r}\,' \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

use to calculate coil fields (use const. current, closed contour)

For conductor element

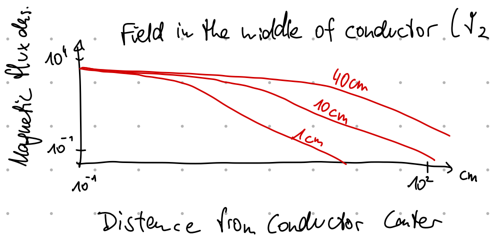


$$\vec{B}(\vec{r}) = \frac{\mu_0 \mu_r}{4\pi} \cdot \frac{I}{s} \cdot (\sin(\vartheta_1) + \sin(\vartheta_2)) \cdot \vec{e}_\varphi$$

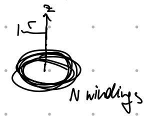
magnetic field in point P

$\vartheta_{1,2} \in [0, +90°]$

Field in the middle of conductor ($\vartheta_2 = -\vartheta_1$)



Field of Short coil on its axis



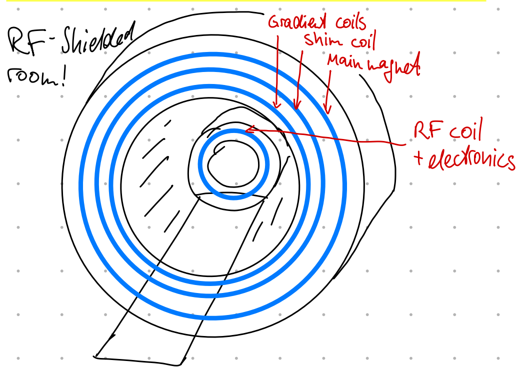
$$\vec{B}(z) = \frac{\mu_0 \mu_r}{2} \cdot N \cdot I \cdot \frac{R^2}{(R^2+z^2)^{\frac{3}{2}}} \cdot \vec{e}_z$$

↳ field strongest at $z=0$:

$$B(0) = \frac{\mu_0 \mu_r}{2} \cdot \frac{N \cdot I}{R} \cdot \vec{e}_z$$

Magnetic resonance imaging

RF-Shielded room!



Wireless Power Transfer Methods

Basically used everywhere

	Capacitive Coupling	Inductive Coupling !	Farfield/Radiative Coupling	Directed Microwave Transmission	Light/Laser Transmission
Freq.	Hz - MHz	Hz - MHz 7-330 MHz used	MHz - GHz	GHz	> THz
Range	< several mm	< several cm 6 in head cavity ok	< several m	< tens of km	< tens of km
Efficiency	High	High	Med	Med/Low	Low
Directivity	Low	Low	Med	High	High
Penetrability	High	High	Med	Low	Low

Directivity: $\frac{\text{max intensity in given direction}}{\text{average intensity}}$

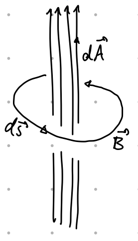
Penetrability: Depth of penetration into material

Magnetic field sources

Generatable by three currents:

- conduction current
- displacement current
- magnetization currents from magnetized media ↳ $\vec{J}_M = \nabla \times \vec{M}$

$$\nabla \times \vec{B} = \underbrace{\mu_0 \overset{\text{conduction current}}{\vec{J}} + \mu_0 \overset{\text{displacement current}}{\frac{\partial \vec{D}}{\partial t}} + \mu_0 \overset{\text{magnetization currents}}{\vec{J}_M}}$$

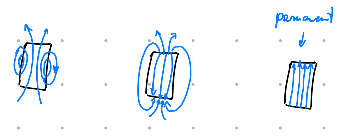


Permanent magnets

Magnetic field strength usually solenoidal
divergence free

but exception: permanently magnetized media

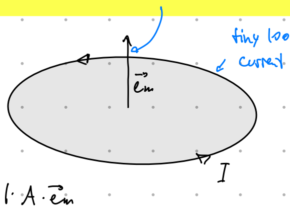
$$\nabla \cdot \vec{H} = -\nabla \cdot \vec{M}$$



$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

SPIN

Magnetic dipole moment



$\vec{m} = I \cdot A \cdot \vec{e}_m$

(ideal dipole: $I \to \infty, A \to 0$)

↳ has magnitude & direction

Calculate magnetic flux density

$$\vec{B}(\vec{r}) = \frac{\mu_0 \mu_r}{4\pi} \cdot \left[\frac{3 \cdot (\vec{m} \cdot \vec{s}) \cdot \vec{s}}{s^5} - \frac{\vec{m}}{s^3} \right]$$

$\longrightarrow \vec{s} = \vec{r} - \vec{r}_m$

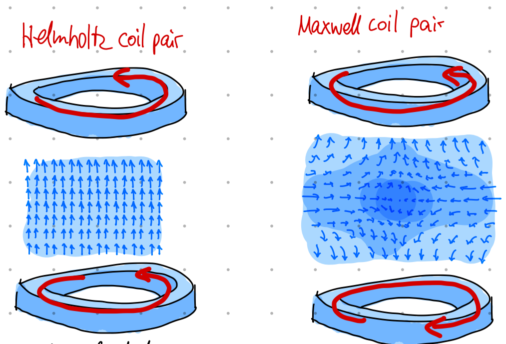
our location, dipole location

Neutron magnetic moment $\mu_n \approx -9.66 \cdot 10^{-27} Am^2$
Proton magnetic moment $\mu_p \approx 1.41 \cdot 10^{-26} Am^2$
Electron magnetic moment $\mu_e \approx -9.28 \cdot 10^{-24} Am^2$ ↳ spin only
Oxygen molecule moment $\mu_{O2} \approx 9.27 \cdot 10^{-24} Am^2$

↳ sometimes magnetons used as units for moments (μ_N, μ_B)

Magnetic particle imaging

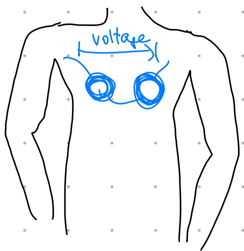
- uses superparamagnetic nanoparticles inside human body (flowing with blood)



- Drive field (25kHz, 25mT)

Selection field (few Hz, 3 $\frac{T}{m}$)

Magnetic Leads



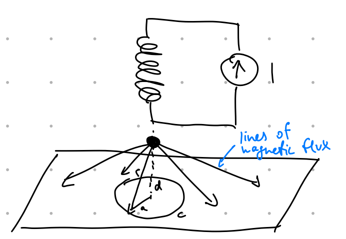
Elementary bipolar lead in magnetic measurement is pair of pick up coils

Lead voltage is induced due to temporal variations of current dipols in body

Pickup coil parameters dependent:

radius vs. height

bigger radius, size better to pick up current density



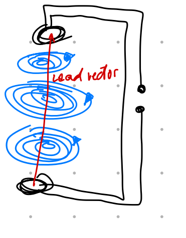
Plane parallel to surface conductor

magnetic lead vector

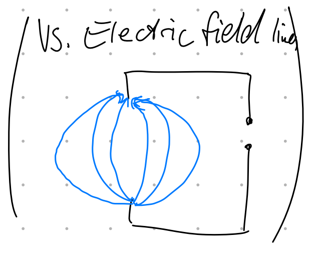
$$\Delta\Phi_{coil} = \vec{p}_i \cdot \vec{L}_{coil}$$

Lead vector field

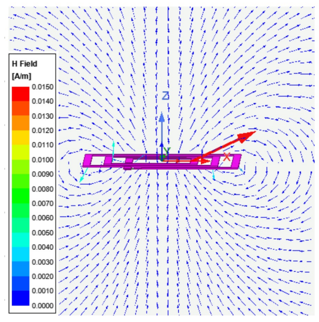
≙ normalized reciprocal current density induced by magnetic field of coil driven by AC current



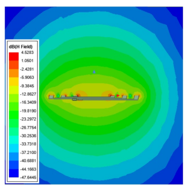
Vs. Electric field lin



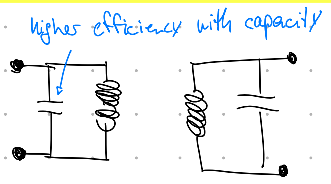
Transmitter magnetic field of transmitter coil



H-field in free space due to 150 mW excitation of [1] computed by a full-wave EM simulator.



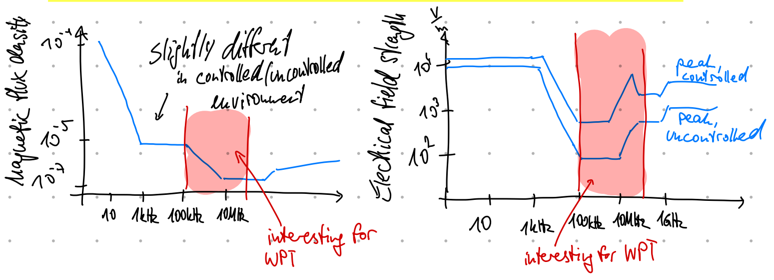
Reason for Resonance



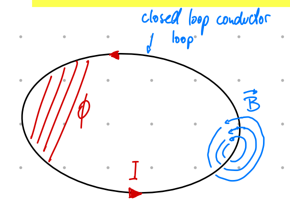
capacitors causes resonant oscillation

⇒ allows high currents without suffering loss!

Maximum permissable exposure limits



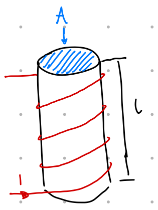
Self Inductance



$$L = \frac{\Phi}{I} = \frac{\iint\limits_{\text{Loop area}} \vec{B} \cdot d\vec{A}}{\iint\limits_{\text{conductor area}} \vec{J} \cdot d\vec{A}}$$

$$\begin{tikzcd} B_{ext}(t) \arrow[r] & E_{ind}(t) \arrow[r] & \vec{J}_{ind} \arrow[ll, bend left, "B_{ind}"] \end{tikzcd}$$

Self Inductance on Solenoid

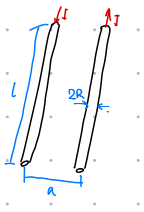


wire-radius ≪ loop radius
loop radius ≪ length l

$$L \approx \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l}$$

number of turns

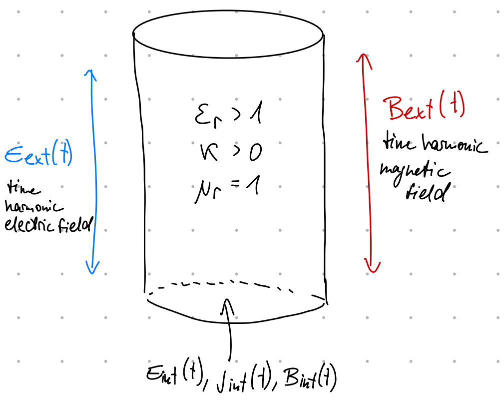
Self inductance on thin wire pair



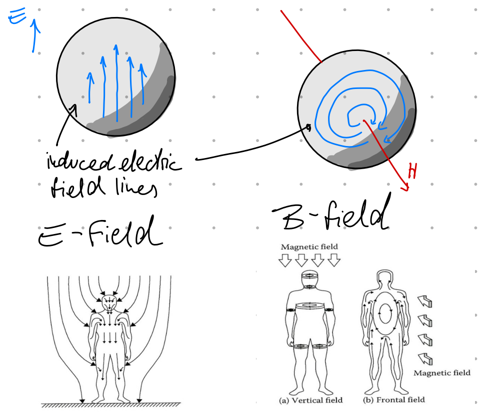
wire radius ≪ wire distance

$$L' = \frac{L}{l} \approx \frac{\mu_0 \mu_r}{\pi} \cdot \ln\left(\frac{a}{R}\right)$$

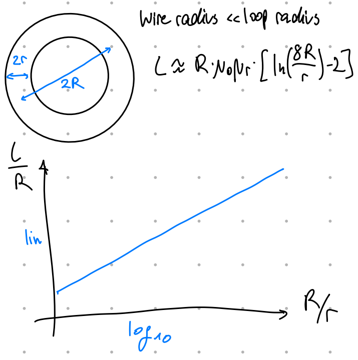
Types of field excitation



Coupling into spherical model



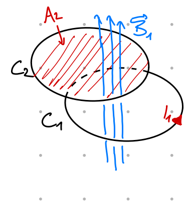
Self inductance on circular loop



Wire radius ≪ loop radius

$$L \approx R \cdot \mu_0 \mu_r \cdot \left[\ln\left(\frac{8R}{r}\right) - 2\right]$$

Mutual imductance



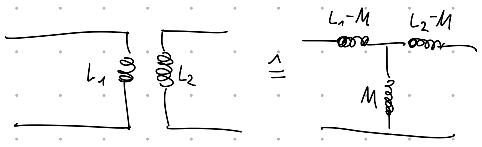
$$L_{21} = \frac{1}{I_1} \cdot \iint\limits_{A_2} \vec{B}_1 \cdot d\vec{A}_2$$

typically $L_{21} = L_{12} = M$

Ratio of mutual to self inductance

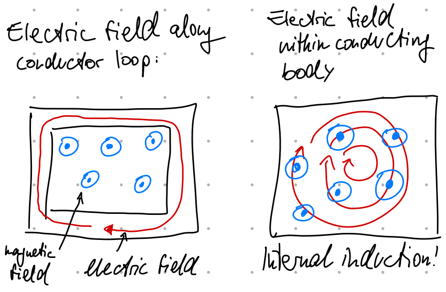
$$k_{1,2} = \frac{M}{L_{1,2}}$$

Equivalent circuit diagrams

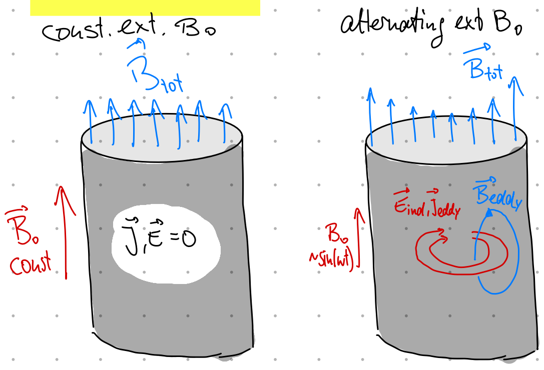


External / Internal Induction

(on time dependant magnetic field)



Eddy current



Alternating ext B_0:

$$\vec{B}_0 = B_0 \cdot \sin(\omega t) \cdot \vec{e}_z$$

$$\downarrow$$

$$\vec{B}_{tot} = \vec{B}_0 + \vec{B}_{eddy}$$

$$\downarrow$$

$$\nabla \times \vec{E}_{ind} = -\frac{\partial \vec{B}_{tot}}{\partial t}$$

$$\downarrow$$

$$\vec{J}_{eddy} = \kappa \cdot \vec{E}_{ind}$$

$$\downarrow$$

$$\nabla \times \vec{B}_{eddy} = \mu_0 \mu_r \cdot \vec{J}_{eddy}$$

Displacement current negligible

Skin depth d_s

If d_s ≫ radius of body R, then low to mid frequency field!

if displacement current negligible:

$$\omega \ll \frac{\kappa}{\varepsilon_0 \varepsilon_r}$$

$$d_s = \sqrt{\frac{2}{\mu_0 \mu_r \cdot \kappa \cdot \omega}} \gg R$$

$$\Downarrow$$

$$\omega \ll \frac{2}{\mu_0 \mu_r \cdot \kappa \cdot R^2}$$

decreases with $\frac{1}{\sqrt{f}}$

Nuclear Magnetic Resonance

- change in orientation of nuclear magnetic moments of a material in a static magnetic field

due to perpendicular, oscillating magnetic field at resonance freq

Magnetization

OR Magnetic Polarization

$$\vec{M} = \Sigma \frac{\vec{m}_V}{V}$$

$\vec{m}_V$ ← magnetic moment in V; V ← small but finite

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

For linear / isotropic media:

$$\Rightarrow \vec{M} = \chi_m \cdot \vec{H}, \quad \chi_m = \mu_r - 1$$

$$= (\mu_r - 1) \cdot \vec{H}$$

Spin

angular momentum $[Nm\,s] = [Js]$

↳ elementary particles have Spin

- unit often divided by Planck's constant $\hbar$
  → dimension-less, "Spin quantum number"

Kern	Spin	$\frac{\mu}{\mu_K}$	$\frac{\gamma}{2\pi}\left(\frac{MHz}{T}\right)$
^{1}H	$\frac{1}{2}$	2.793	42.573
^{2}D	1	0.857	6.535
^{13}C	$\frac{1}{2}$	0.7024	10.705
^{23}Na	$\frac{3}{2}$	2.2175	11.263
^{29}Si			
^{31}P			

Only nuclei with odd atomic mass no possess a net spin!

Magnetic Moment

Gyromagnetic ratio

$$\boxed{\vec{m} = \gamma \cdot \vec{S}}$$

$$\gamma \left[\frac{m^2}{Vs^2}\right] = \left[\frac{Hz}{T}\right]$$

	H	C	N	O	Na	P
$\gamma \left[\frac{MHz}{T}\right]$	42.57	10.7	3.08	5.772	11.26	17.25
Atomic mass number	1	13	15	17	23	31

Interaction with static B-Fields

Strong, static B-Field (B0 field in NMR) changes orientation of spin of nuclei, interacts with nuclei's magnetic moments

↳ Found in macroscopic level! On quantum mechanical level: only discrete, equidistant levels

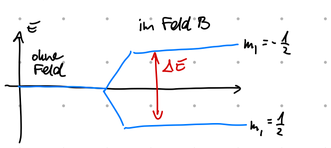
amount of change if B-field in z-direction

$$\Delta m_z = \gamma \cdot \hbar$$

corresponding change of energy

$$\Delta E = \Delta m_z \cdot B_0 = \gamma \hbar \cdot B_0$$

Energieniveau eines Kerns mit Drehimpuls $J = \frac{1}{2}$



Maxwell-Boltzmann Statistic

Magnetic moments over two energy states follow Maxwell-Boltzmann statistic in thermal equillibrium

Probability p to find state at room temperature:

$$\frac{P_{parallel}}{P_{antiparallel}} = e^{\frac{\Delta E}{k_B T}} \approx 1 + \frac{\Delta E}{k_B T} \quad (\Delta E \ll k_B T)$$

Only small number of spins more parallel than anti-parallel!

$$P_{parallel} - P_{anti\text{-}parallel} \approx \frac{\Delta E}{2 k_B \cdot T} \quad \Delta E \ll k_B T$$

Spacial encoding

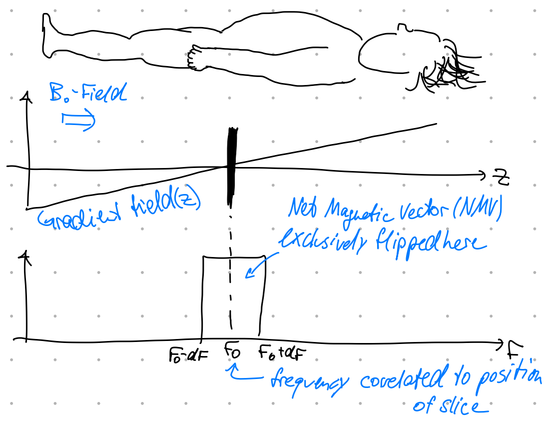
↳ get slices of patient

Use magnetic field with gradient in z-direction

Superimposing B0-Field,

Strength of B-field corresponds to shift in resonance frequency

RF-frequency then sweeps through all possible resonance frequencies and therefore slices in space



Interaction with RF B-Fields

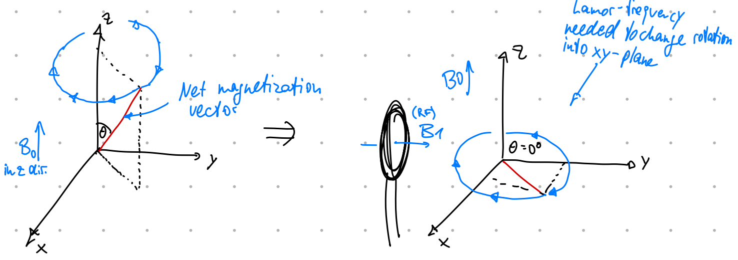
RF magnetic field (B1-Field) superposed onto B0-field capable of changing distribution of nuclei over discrete energy levels

$$\Delta E = \hbar \cdot \omega$$

specific frequency needed

$$\omega_0 = \gamma \cdot B_0$$

"Lamor frequency"



nur B0-Feld

B0-Feld mit B1-Feld orthogonal

Pulsed MRI

(B1-field is pulsed)

- TP: Pulse time (duration of pulse)
  ↳ determines how much magnetization will flip from main orientation
  flip analge $\Theta \approx \gamma \cdot B_1 \cdot TP$
- TR: repitition time (time between pulses)
- TE: Echo time (between pulse & response)

Relaxation in MRI

(after B1 turned off)

↳ magnetization returns to equillibrium

- T1: spin-lattice / longitudinal relaxation (z-direction)
- T2: spin-spin / transversal relaxation

→ T1 & T2 constants depend on electron density distribution in orbitals surrounding the nucleus ⇒ distinguish molecules

- After one 90° Pulse: "Free induction decay (FID)"
- After two 90° Pulses: "Spin echo (SE)"

Simulate using CONCEPT II by TUHH

SAR in MRI

1. RF magnetic field induces AC current in body:

$$\text{rot}\, \underline{\vec{E}} = -j\omega\mu_0\mu_r \cdot \underline{\vec{H}}$$

2. Alternating $\vec{E}$-field leads to ohmic losses:

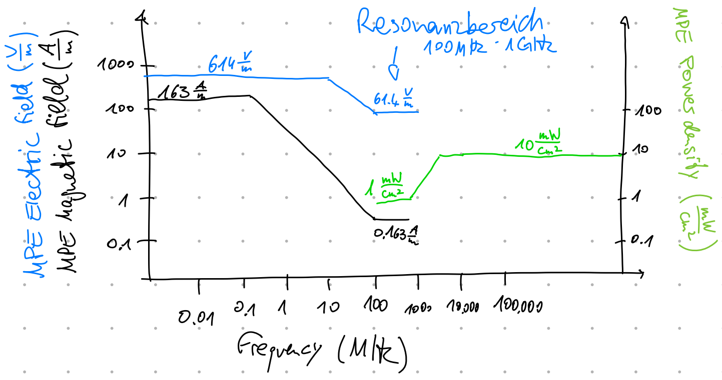
Power loss density: $p = \frac{1}{2} \cdot \kappa'_{equiv} \cdot |\underline{\vec{E}}|^2$

3. specific absorption rate $SAR = \frac{1}{2} \cdot \frac{p}{\rho_{mass}}$

4. SAR over space:

$$SAR_m = \frac{1}{V_m} \cdot \iiint SAR\, dV$$

Maximum Permissible Exposure Limit

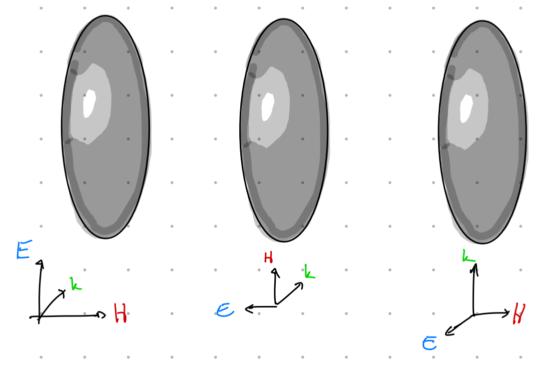


With rising frequency

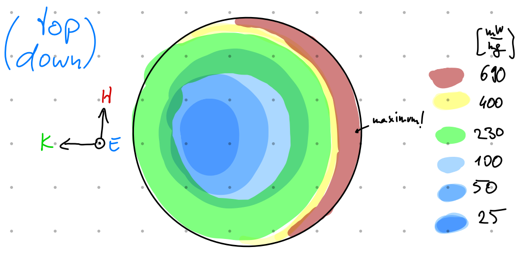
1. E-Field MPEL sinks to $61.4 \frac{V}{m}$ (10%) at over 100 MHz
   H-Field MPEL sinks to $0.163 \frac{A}{m}$ (1%) at over 100 MHz
2. Only Power density MPEL defined > 1 GHz, rises from $1 \frac{mW}{cm^2}$ to $10 \frac{mW}{cm^2}$

Power MPEL in many countries at $0.01 \frac{mW}{cm^2}$! (0.3-300 GHz)

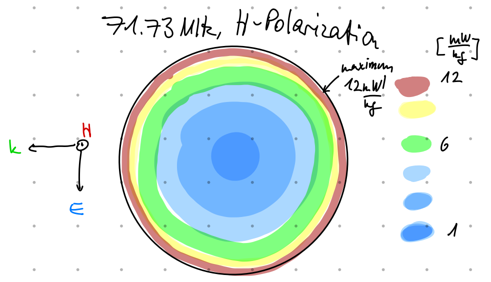
Plane wave polarizations



71.73 MHz, E-Polarization SAR

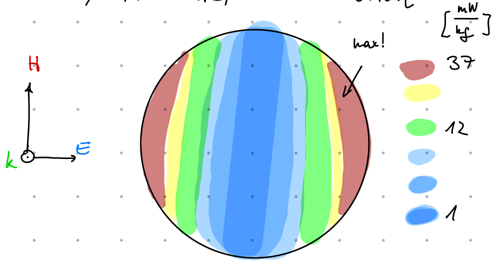


71.73 MHz, H-Polarization



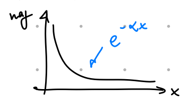
↳ a lot less SAR

71.73 MHz, K-Polarization



Effects of losses in media

- exponential attenuation
- generation of power loss density
- reduction of phase velocity / wavelength
- introduction of phase offset between B & E fields



- conductive / ohmic losses
- dielectric losses
- magnetic losses

Poynting vector reduction

$$\underline{\vec{S}} = \underline{S}_0 \cdot e^{-2\alpha x} \cdot \vec{e}_x$$

Power loss density

$$p = -Re\{\nabla \cdot \underline{\vec{S}}\} = 2\alpha \cdot Re\{\underline{S}_0\} \cdot e^{-2\alpha x}$$

Poynting Vector

$$\underline{\vec{S}} = \frac{1}{2} \cdot \underline{\vec{E}} \times \underline{\vec{H}}^* \left[\frac{W}{m^2}\right]$$

Sonnenlicht: $140 \frac{mW}{cm^2}$

For plane waves in free space / air: Poynting vector is real & represents time-averaged flux of power per area (Power density)

MPEL: $1\text{-}10 \frac{mW}{cm^2}$

Power loss density

$$p = \frac{1}{2} \cdot \kappa'_{equiv} \cdot |\underline{\vec{E}}|^2 \left[\frac{W}{m^3}\right]$$

ohmic loss

Specific absorption rate (SAR)

$$SAR = \frac{p}{\rho_{mass}} = \frac{1}{2} \cdot \frac{\kappa'_{equiv} \cdot |\underline{\vec{E}}|^2}{\rho_{mass}} \quad \left[\frac{W}{kg}\right]$$

normalization of power loss density with local mass density

$\frac{1}{2}$ does not appear always!

↳ can be integrated over volume MPEL × 0.08-0.4 $\frac{W}{kg}$

$$SAR_m = \frac{1}{V_m} \cdot \iiint SAR \, dV$$

corresponding to mass m (e.g. 1g, 10g, whole body)

Example: When holding 902 MHz cellular mobile telephone next to head: biggest values 1.25 $\frac{W}{kg}$

↳ varies between 0.44 & 3.6 W (depends on antenna type, worse with frequency)

↳ For human body: Peaks at 30-300 MHz!

Poynting to dT/dt

$$\underline{\vec{S}} = \frac{1}{2} \cdot \underline{\vec{E}} \times \underline{\vec{H}}^*$$

(Power loss density

$$p = \frac{1}{2} \kappa'_{equiv} \cdot |\underline{\vec{E}}|^2$$

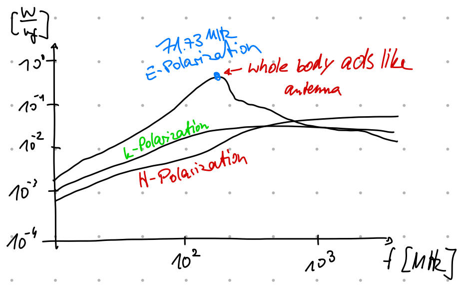
| / Massendichte

$$SAR = \frac{p}{\rho_{mass}}$$

↓ / Wärmekapazität

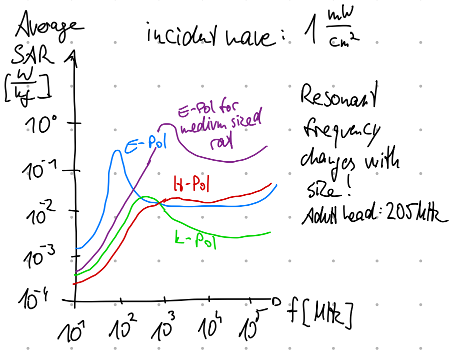
$$\frac{dT}{dt} = \frac{SAR}{c} = \frac{p}{c \cdot \rho_{mass}}$$

SAR for water ellipsoid



Whole-Body SAR I

incident wave: $1 \frac{mW}{cm^2}$



Resonant frequency changes with size! Adult head: 205 MHz

Conductive losses

assume complex wave number: $\underline{k} = \sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r\omega^2 - j\mu_0\mu_r\kappa\omega}$

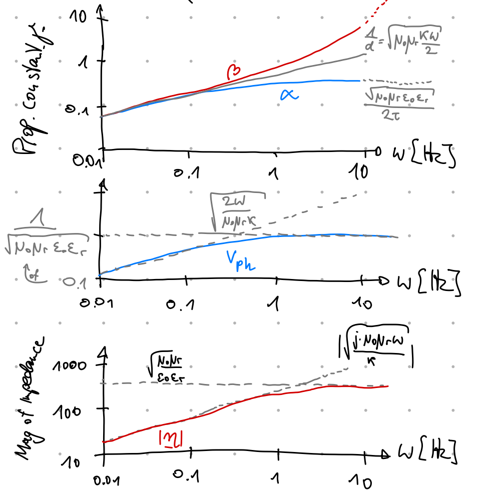
$$\underline{\gamma} = \alpha + j\beta = j\underline{k}$$

$$\Rightarrow \alpha = \omega \cdot \sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}\left(\sqrt{1+\frac{1}{\omega^2\tau^2}} - 1\right)}$$

$$\beta = \omega \cdot \sqrt{\frac{\mu\varepsilon_0\varepsilon_r}{2}\left(\sqrt{1+\frac{1}{\omega^2\tau^2}} + 1\right)}$$

$\tau = \frac{\varepsilon}{\kappa}$ relaxation time

$$\tau = \frac{\varepsilon_0\varepsilon_r}{\kappa} = 1s \text{ here!}$$



SAR & Heat

$$\frac{dW_{heat}}{dt} = c \cdot m \cdot \frac{dT}{dt}$$

heat capacity, mass, tissue temperature

$c_{water} = 4000 \frac{J}{kg \cdot K}$

Assume all power loss leads to heat:

$$SAR = \frac{P}{\rho_{mass}} = \frac{1}{m} \cdot \frac{dW_{heat}}{dt}$$

$$\Rightarrow \frac{dT}{dt} = \frac{SAR}{c}$$

← spezifische Wärmekapazität

SAR = 0.4 $\frac{W}{kg}$, dt = 1h, 1 litres of water ⇒ 0.36 K

Heat capacity + density

	Specific Heat capacity $(\frac{J}{kg\,°C})$ celsius	Density $(\frac{kg}{m^3})$
Skeletal muscle	3470	
Fat	2260	940
Bone, cortical	1260	1790
Bone, spongy	2970	1250
Blood	3890	1060
Water	4180	1000

Plane wave propagation

- requires fully dynamic electromagnetic fields
  ↳ magnetic & displacement current fields need to be time dependent
- time & space tightly coupled!
- harmonic wave speed in free space:

$$c_0 = \lambda \cdot f = \frac{\lambda_0}{T} = 299.792458 \frac{m}{s}$$

Plane wave

"Transverse electromagnetic" TEM

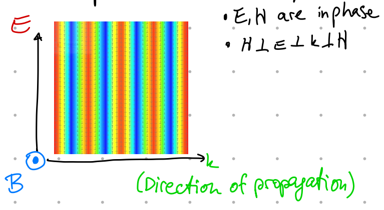
- vector field of const. freq with
  - infinite parallel planes as wavefronts
  - propagates into direction perpendicular to wavefronts

→ solves Helmholtz & Maxwell's equations

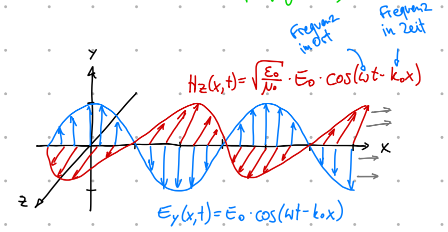
$\dot{H}, \dot{B} \rightarrow E$ induced

$C_H \leftarrow \dot{D}, \dot{E}$

In free space: E- & H-Field perpendicular to direction



- E, H are in phase
- H ⊥ E ⊥ k ⊥ H



Kreiswellenzahl → $k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0} \left[\frac{1}{m}\right]$

or $\vec{E}(\vec{r},t) = \underline{\vec{E}}_0 \cdot e^{j(\omega t - \underline{\vec{k}} \cdot \vec{r})}$

Linear Polarization

E & H-Field oscillation in fixed planes in space

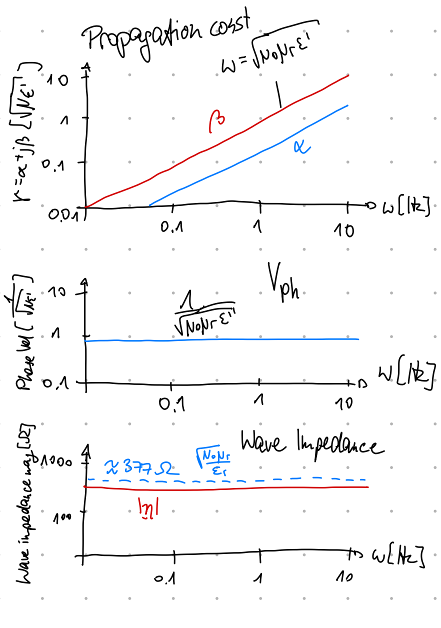
$\Rightarrow \underline{\vec{E}}_0 = \vec{E}_0 \cdot e^{j\varphi}$ ← space independent

Dielectric losses

$$\varepsilon_0\varepsilon_r = \varepsilon' - j\varepsilon'' \quad \kappa = 0$$

$$\alpha = \omega \cdot \sqrt{\frac{\mu_0\mu_r\varepsilon'}{2}\left(\sqrt{1+\tan(\delta_e)^2} - 1\right)}$$

$$\beta = \omega \cdot \sqrt{\frac{\mu_0\mu_r\varepsilon'}{2} \cdot \sqrt{1+\tan(\delta_e)^2} + 1}$$



Superposition

$$\underline{\vec{E}}_1 = \underline{\vec{E}}_{01} \cdot e^{j(\omega_1 t - \underline{\vec{k}}_1 \vec{r})}$$

$$\underline{\vec{E}}_2 = \underline{\vec{E}}_{02} \cdot e^{j(\omega_2 t - \underline{\vec{k}}_2 \cdot \vec{r})}$$

$\vec{E}_1 + \vec{E}_2$ depends on: Amplitude vector, freq., direction

⇒ Interference

Example: $\underline{\vec{E}}_1 = \vec{E}_{01} \cdot e^{j\varphi}$

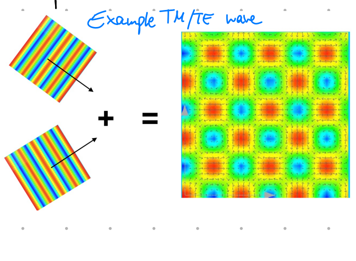
$\underline{\vec{E}}_2 = \vec{E}_{02}$

$\vec{E}_{01} \perp \vec{E}_{02}$ → E-fields perpendicular! → two different polarizations!

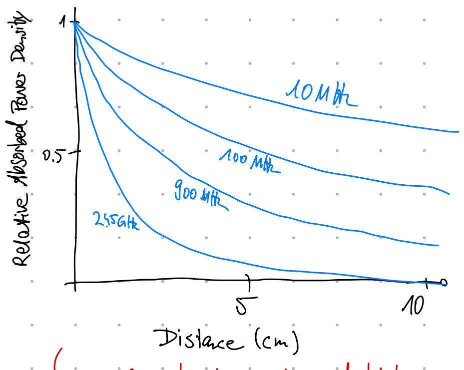
Then superposition will be

- again a plane wave
- linear polarization $\varphi = 0° \vee \varphi = 180°$
- circular polarization $E_{01} = E_{02} \wedge \varphi = \pm 90°$
- elliptic polarization none of above

Example TM/TE wave



Power density distribution in muscle



↳ significant attenuation at high frequencies!

Frequencies & Wavelength

	Freq	Period	Wavelength
Power Transmission	50 Hz	20 ms	6000 km
VHF Radio	100 MHz	10 ns	3 m
cell phones	2 GHz	500 ps	15 cm
Satellite TV	10 GHz	100 ps	3 cm
Fast scope edge	100 GHz	10 ps	3 mm
Green light	600 THz	1.7 fs	500 nm

Body wavelength

	$\frac{\lambda_0}{\lambda}$
400 MHz	7.9
900 MHz	7.5
5.8 GHz	7.1

factor 7-8 smaller!

Wave properties

Complex wave number

$$\underline{k} = \underline{k}(\omega) = \sqrt{\mu\varepsilon\omega^2 - j\mu\kappa\omega}$$

attenuation per unit distance

↳ dispersion relation

→ Frequenz im Ort kann abhängig von Frequenz in Zeit sein!

$$\varepsilon_0\varepsilon_r = \varepsilon' - j \cdot \varepsilon'', \ \kappa = 0$$

$$\Rightarrow \underline{k} = \sqrt{\mu_0\mu_r\varepsilon'\omega^2 - j\mu_0\mu_r\varepsilon''\omega^2}$$

if no losses: $\underline{k} = k = \omega \cdot \sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$

Propagation constant

$$\underline{\gamma} = \alpha + j\beta = j \cdot \underline{k}$$

$$\alpha = \omega \cdot \sqrt{\frac{\mu_0\mu_r\varepsilon'}{2}\left(\sqrt{1+(\tan(\delta_e))^2} - 1\right)}$$

$$\beta = \omega \cdot \sqrt{\frac{\mu_0\mu_r\varepsilon'}{2} \cdot \left(\sqrt{1+(\tan(\delta_e))^2} + 1\right)}$$

$\tan(\delta_e) = \frac{\varepsilon''}{\varepsilon'}$ electric loss tangent

wird groß wenn κ >> ε, μ groß

Attenuation Constant

$$\alpha(\omega) = Re\{j \cdot \underline{k}(\omega)\}$$

↳ 0 wenn ε >> κ

Phase constant

$$\beta(\omega) = Im\{j \cdot \underline{k}(\omega)\} \left[\frac{1}{m}\right]$$

Wavelength

$$\lambda(\omega) = \frac{2\pi}{\beta(\omega)}$$

Phase velocity

$$v_{ph}(\omega) = \frac{\omega}{\beta(\omega)} = \lambda \cdot f$$

Complex wave impedance

$$\underline{\eta} = \underline{\eta}(\omega) = \frac{j\underline{k}(\omega)}{\kappa + j\varepsilon\omega} = \frac{\mu \cdot \omega}{\underline{k}(\omega)} = \sqrt{\frac{j\mu\omega}{\kappa + j\varepsilon\omega}}$$

↑ if no conduction: not freq.-dependent!

also common: $\underline{\Gamma}$, $\underline{Z}_W$

impedance in free space / air

In lossless media:

$$\underline{\eta} = \eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \eta_0, \quad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 376.73\,\Omega$$

In metals

$$\underline{\eta} = (1+j) \cdot \sqrt{\frac{\mu\omega}{2\kappa}} = \frac{1+j}{\kappa d_s}$$

(Surface impedance $\underline{Z}_s$)

Skin depth

$$d_s = \sqrt{\frac{2}{\mu\kappa\omega}}$$

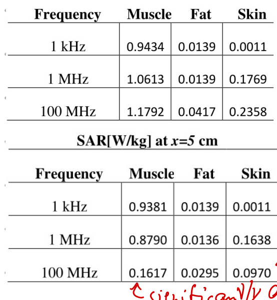
Complex index of refraction

$$\underline{n} = \underline{n}(\omega) \equiv \frac{\underline{k}(\omega)}{k_0(\omega)}$$

In lossless, freq. indep. media:

$$n = \frac{2\pi/\lambda}{2\pi/\lambda_0} = \frac{\lambda}{\lambda_0} = \frac{c_0}{v_{ph}} = \sqrt{\mu_r\varepsilon_r}$$

SAR at 50V/m



SAR[W/kg]

Frequency	Muscle	Fat	Skin
1 kHz	0.9434	0.0139	0.0011
1 MHz	1.0613	0.0139	0.1769
100 MHz	1.1792	0.0417	0.2358

Boundary

SAR[W/kg] at x=5 cm

Frequency	Muscle	Fat	Skin
1 kHz	0.9381	0.0139	0.0011
1 MHz	0.8790	0.0136	0.1638
100 MHz	0.1617	0.0295	0.0970

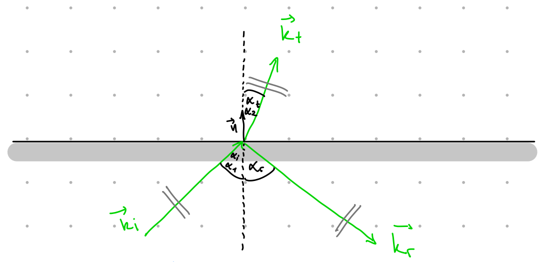
similar but high absorbed power 5 cm inside

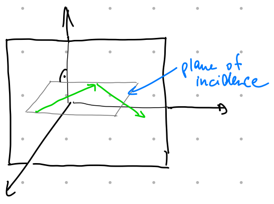
← -40%

↑ significantly decreased

temperature increase: muscle, border, 1 kHz: $0.25 \frac{dT}{dt} [10^{-3} \frac{°C}{s}]$

Plane Wave Reflection & Refraction





Wellenzahl!

$\underline{k}_i = \underline{k}_r = \underline{\underline{k_1}}, \quad \underline{k}_t = \underline{\underline{k_2}}$

$\alpha_i = \alpha_r = \underline{\underline{\alpha_1}} \quad \alpha_t = \alpha_2$ Reflection law

$\underline{k}_1 \cdot \sin(\alpha_1) = \underline{k}_2 \cdot \sin(\alpha_2)$ Refraction law → Snell's law

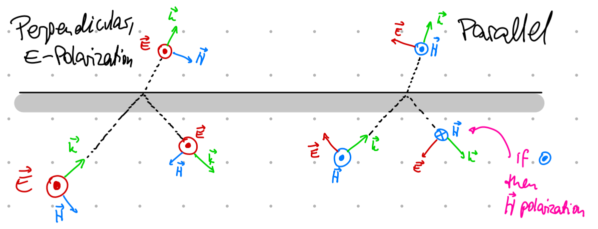
Grenzwinkel für totale Reflexion

$\sin(\alpha_1) = \frac{k_2}{k_1} \quad (k_2 < k_1, \alpha_2 = 90^\circ)$

$\underline{k}_{1,2} = \sqrt{\mu_{1,2} \cdot \varepsilon_{1,2} \omega^2 - j \cdot \mu_{1,2} \cdot \kappa_{1,2} \cdot \omega}$

proportional field amplitude rations, not power ratios!

Perpendicular & Parallel Polarization



Fresnel's Formulas

For E-Polarization

→ relate space independent part of complex amplitudes of reflected & transmitted E-field to space independent part of incident E-field:

$$\underline{r}_E = \left(\frac{\underline{E}_{0r}}{\underline{E}_{0i}}\right) = \frac{\underline{\eta}_2 \cdot \cos(\alpha_1) - \underline{\eta}_1 \cdot \cos(\alpha_2)}{\underline{\eta}_2 \cos(\alpha_1) + \underline{\eta}_1 \cdot \cos(\alpha_2)}$$

$$\underline{t}_E = \left(\frac{\underline{E}_{0t}}{\underline{E}_{0i}}\right) = \frac{2 \underline{\eta}_2 \cdot \cos(\alpha_1)}{\underline{\eta}_2 \cdot \cos(\alpha_1) + \underline{\eta}_1 \cdot \cos(\alpha_2)}$$

where $\underline{\eta}_{1,2} = \sqrt{\frac{j \cdot \mu_{1,2} \cdot \omega}{\kappa_{1,2} + j \cdot \varepsilon_{1,2} \cdot \omega}}$

For H-Polarization

→ … magnetic fields

$$\underline{r}_H = \left(\frac{\underline{H}_{0r}}{\underline{H}_{0i}}\right) = \frac{\underline{\eta}_1 \cdot \cos(\alpha_1) - \underline{\eta}_2 \cdot \cos(\alpha_2)}{\underline{\eta}_1 \cdot \cos(\alpha_1) + \underline{\eta}_2 \cdot \cos(\alpha_2)}$$

$$\underline{t}_H = \left(\frac{\underline{H}_{0t}}{\underline{H}_{0i}}\right) = \frac{2 \cdot \underline{\eta}_1 \cdot \cos(\alpha_1)}{\underline{\eta}_1 \cdot \cos(\alpha_1) + \underline{\eta}_2 \cdot \cos(\alpha_2)}$$

$$\underline{\eta}_{1,2} = \sqrt{\frac{j \cdot \mu_{1,2} \cdot \omega}{\kappa_{1,2} + j \cdot \varepsilon_{1,2} \cdot \omega}}$$

Special cases

$\alpha_1 = 0^\circ$

$$\underline{r}_E = -\underline{r}_H = \frac{\underline{\eta}_2 - \underline{\eta}_1}{\underline{\eta}_2 + \underline{\eta}_1} \quad \underline{t}_E = \frac{2\underline{\eta}_2}{\underline{\eta}_2 + \underline{\eta}_1} \quad \underline{t}_H = \frac{2 \cdot \underline{\eta}_1}{\underline{\eta}_2 + \underline{\eta}_1}$$

Tangential incidence:

$\alpha_1 = 90^\circ \quad \underline{r}_E = \underline{r}_H = -1 \quad \underline{t}_E = \underline{t}_H = 0$

Incidence on perfect conductor $(\underline{\eta}_2 = 0)$

$\underline{r}_E = -\underline{r}_H = -1 \quad \underline{t}_E = 0 \quad \underline{t}_H = 2$

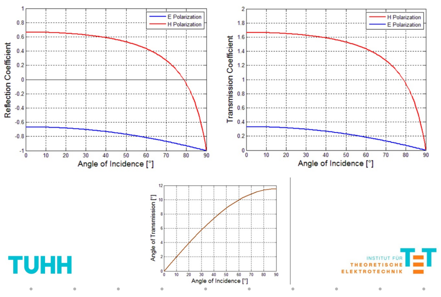
Loss free dielectrics

→ use refractive index:
(instead of wave impedance/wave number)

$n = \frac{c_0}{v_{ph}} = \sqrt{\mu_r \varepsilon_r}$

$k = \frac{\omega}{v_{ph}} = \frac{\omega}{c_0} \cdot n, \quad \eta = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = v_{ph} \cdot \mu_0 \mu_r = \frac{c_0}{n} \cdot \mu_0 \mu_r$

Example: n = 1 → n = 5



Lossy E/H Field distribution

Hard to predict! Need simulator

→ in general: after 2 cm inside body, $\frac{E_{inside}}{E_{outside}} < 0.2$

→ weird behavior, SAR too

→ magnetic fields penetrate more

Near field vs Far field

distance from antenna below wavelength

distance bigger than $\underline{\underline{2}}$ wavelengths

Numerical methods

- BEM: Boundary Element Method
- FDM: Finite Difference method
- FDTD: Finite Difference Time Domain
- FEM: Finite Element Method
- MoM: Method of Moments

- volume vs surface
- time vs. freq. domain
- asymptotic vs. full wave
- implicit vs. explicit
- differential vs. integral

→ concept 2: surface, freq. domain, implicit, integral

Conductive losses

$\underline{k} = \sqrt{\mu \varepsilon \omega^2 - j \mu \kappa \omega}$

$\underline{\gamma} = \alpha + j\beta = j \cdot \underline{k}$

$\Rightarrow \alpha + j\beta = j \cdot \sqrt{\mu \varepsilon \omega^2 - j \mu \kappa \omega} \quad |()^2$

$\alpha^2 - \beta^2 + j 2\alpha\beta = j \mu \kappa \omega - \mu \varepsilon \omega^2$

I $\quad \alpha^2 - \beta^2 = -\mu \varepsilon \omega^2 \Rightarrow \alpha = \sqrt{-\mu \varepsilon \omega^2 + \beta^2}$

II $\quad 2\alpha\beta = \mu \kappa \omega$

$2\sqrt{-\mu \varepsilon \omega^2 + \beta^2} \cdot \beta = \mu \kappa \omega$

$4(-\mu \varepsilon \omega^2 + \beta^2) \cdot \beta^2 = \mu^2 \kappa^2 \omega^2$

$-4 \mu \varepsilon \omega^2 \beta^2 + 4\beta^4 = \mu^2 \kappa^2 \omega^2$

$\beta^4 - \mu \varepsilon \omega^2 \beta^2 = \frac{1}{4} \mu^2 \kappa^2 \omega^2$

$\beta^4 \underbrace{- \mu \varepsilon \omega^2}_{p} \beta^2 \underbrace{- \frac{1}{4} \mu^2 \kappa^2 \omega^2}_{q} = 0$

$$\beta^2_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$= \frac{\mu \varepsilon \omega^2}{2} \pm \sqrt{\left(\frac{\mu \varepsilon \omega^2}{2}\right)^2 + \frac{1}{4} \mu^2 \kappa^2 \omega^2}$$

$$= \omega^2 \cdot \frac{\mu \varepsilon}{2} \pm \sqrt{\frac{1}{4} \cdot \mu^2 \varepsilon^2 \omega^4 + \frac{1}{4} \mu^2 \kappa^2 \omega^2}$$

$$= \omega^2 \cdot \frac{\mu \varepsilon}{2} \pm \sqrt{\frac{1}{4} \mu^2 \omega^4 \left(\varepsilon^2 + \frac{\kappa^2}{\omega^2}\right)}$$

$$= \omega^2 \cdot \frac{\mu \varepsilon}{2} \pm \frac{1}{2} \mu \omega^2 \sqrt{\varepsilon^2 + \frac{\kappa^2}{\omega^2}}$$

let $\tau = \frac{\varepsilon_0 \varepsilon_r}{\kappa}$

$\Rightarrow \kappa^2 = \frac{\varepsilon^2}{\tau^2}$

$$= \omega^2 \left(\frac{\mu \varepsilon}{2} \pm \frac{1}{2} \mu \sqrt{\varepsilon^2 + \frac{\varepsilon^2}{\tau^2 \omega^2}}\right)$$

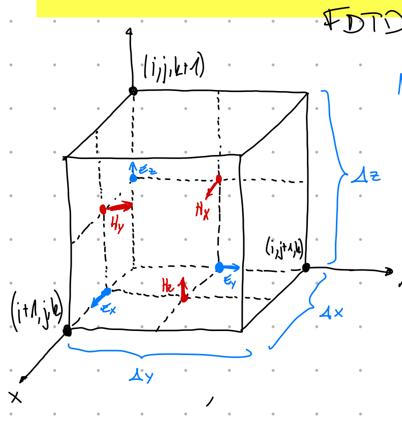
$$= \omega^2 \left(\frac{\mu \varepsilon}{2} \pm \frac{1}{2} \mu \varepsilon \cdot \sqrt{1 + \frac{1}{\tau^2 \omega^2}}\right)$$

$$= \omega^2 \left(\frac{\mu \varepsilon}{2} \left(1 \pm \sqrt{1 + \frac{1}{\tau^2 \omega^2}}\right)\right)$$

$$\beta_{1/2} = \omega \cdot \sqrt{\frac{\mu \varepsilon}{2} \left(\sqrt{1 + \frac{1}{\tau^2 \omega^2}} + 1\right)}$$

Finite Difference Time Domain

FDTD

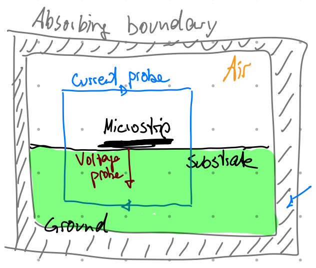


Model
1. Generalize discrete model
2. Adding absorption boundaries, probes

3. Inject wave/excitation of feed
4. Record EM fields & derived statistics
5. Simulation until scattering complete

6. Inspection of fields & derived quantities in time domain

7. Further param extraction after FFT



- get more detailed impedance/freq. characteristics

Perfectly matched layer, totally absorbing

Algorithm:

$\nabla \times H = \varepsilon_0 \varepsilon_r \cdot \frac{\partial \vec{E}}{\partial t}$ I $\quad \nabla \times \vec{E} = -\mu_0 \mu_r \cdot \frac{\partial \vec{H}}{\partial t}$ II

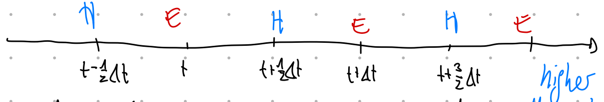
annähern
$-\frac{dH}{dx} = \varepsilon_0 \varepsilon_r \cdot \frac{dE_y}{dt} \quad \frac{dE_y}{dx} = \mu_0 \mu_r \frac{H_z}{dt}$

⇓

I $\quad E_z(t + \Delta t) = E_z(t) + \frac{\Delta t}{\varepsilon_0 \varepsilon_r} \cdot f(H_x, H_y)$ ← in 2D

↑ given at $t + \frac{1}{2}\Delta t$

II $\quad H_{x,y}\left(t + \frac{3}{2}\Delta t\right) = H_{x,y}\left(t + \frac{1}{2}\Delta t\right) - \frac{\Delta t}{\mu_0 \mu_r} \cdot \overset{?}{g}_{x,y}(E_z)$



- grid spacing needs to be chosen that
  - → feature resolution of model is good
  - → Nyquist sampling theorem is met $\max(\Delta x, \Delta y, \Delta z) \leq \frac{\min(\lambda_{signal})}{2}$
  - → in practice more strict!
- Time step needs to be chosen that
  - → CFL stability criterion
  - → Nyquist sampling criterion $\max(f_{signal}) \leq f_{Ny} = \frac{1}{2\Delta t}$
- errors: due to modelling, discretization, implementation, round off

higher grid sampling allows lower phase velocity error (inverse exponential)

THz Waves, Infrared, Visible light

- Waves → Rays Models, switch at 10THz
- Millimeter waves max: 300GHz: 1mm
- THz waves max: 10 THz: 30μm
- Infrared max: 400THz 750nm
- Visible light max: 800THz 375 nm
- UV max: 3PHz 100nm

(min 750nm)

Effects (additional)

- multiple scattering
- absorption!
- specular reflection
- negligible interference
- Increased spectral bandwidth

- human body: Mostly opaque "lichtundurchlässig"

Geometrical optics

- approximative model of wave propagation
- in dielectrics of slowly varying refractive index

$\frac{\Delta n}{n} \ll 1$

- Waves described by rays
- rays ≙ gradients of surfaces of constant phase

---

at interface to electrical conductor:

specular reflection

at interface to dielectric:

$n_1 \cdot \sin(\alpha_1) = n_2 \cdot \sin(\alpha_2)$ (Snell's Law)

Coherence & Interference

- two EM waves show coherence, if
  - → same center frequency
  - → constant phase difference over ∞ time

⇒ allows temporally & spatially persistant interference effects

→ coherence time is reduced, if coherence not perfect

$\tau \approx \frac{1}{\Delta f}$

Optical coherence tomography

?

THz Environment

- max frequency of electronic solid state sources: ≈ 2THz
- min frequency of laser sources: 0.5 THz
- skin properties: with rising freq:
  - → real index of refraction decreases
  - → absorption coeff. increases
  - → optical penentration depth decreases
- less water → higher peatration depth
- thermal effect: exposure duration + temp result in cell dygradation
- THz Exposure Standard: multiple at different levels, $0.01\ \frac{W}{m^2}$ to ≈1 $\frac{W}{m^2}$ later

Optical properties of skin

mit größerer Frequenz: (wahrnehmbares Licht)

- Absorption nimmt zu
- Streuung nimmt zu
- g-Faktor nimmt ab

Reflektion ist wie Low-Pass Filter auf Eingangs-Dirak-Puls

Thermal radiation

- multitude of EM waves
- generated by thermal motion of charged particles
  ↳ fundamental for heat transfer
- a lot of different laws for this
- quantization: intensity

---

human: with rising external temperature

→ heat radiation decreases

→ convection decrease

→ verdunstung decreases

Peak black body radiation at ≈ 35THz

Radiative transfer

↳ if radiation passes through medium

- looses intensity due to absorption
- redistributes intensities by scattering
- gains intensity by emission

Simplified, 1D form: Lambert-Beer-Law

$I(x) = I_0 \cdot e^{-\mu_a x} \cdot e^{-\mu_s x}$

↖ attenuation due to absorption/scattering

absorbed power density:

$p(x) = \mu_a \cdot I(x)$

UV & Xrays

→ 2PHz

↳ Photon model!

UV: 800THz - 3 PHz

XRays: 3PHz-60 EHz

→ ionizing effects! Starting at 2PHz

Planchsches Wirkungsquantum

$E_{photon} = h \cdot f$ → 2PHz → 8eV ⇒ enough to liberate H electron

$1eV = 1.602 \cdot 10^{-19} J$

XRays max. 250keV (60EHz)

- ionization energy decreases with atomic number
- a lot of different interactions between
  - atom cores
  - electrons
  - photons

⇒ Photoeffekt

Klassische Streuung

Compton-Streuung

Paarbildung

Kernreaktion

- With increasing frequency / energy ...
  1. Photoeffect dominate ← attenuation
  2. Compton effect dominate
  3. Pair production dominate
- absorption coefficient $\mu \approx Z^{3.4}$ $[\frac{1}{m}]$
  ↳ Pb absorbes everything after a few wavelengths
- human body mostly transparent, exception bones

Human effects

- Deterministic:
  - acute radiation syndrome (ARS)
    - → nausea, diarrhea, headache, skin burns, fever
- Stochastic: cancer, genetic defects, mutation

Ionizing radiation quantities

- Ionization dose: $J = \frac{\text{Ionized Charge}}{\text{Mass}}$ $[\frac{C}{kg}]$
- Energy dose/absorbed dose: $D = \frac{\text{Absorbed Energy}}{\text{Mass}}$ $[\frac{J}{kg} = Gy]$
- Equivalent dose: $H = Q \cdot D$ $(\frac{J}{kg} = Sv)$

Quality factor = 1 for photons

Generation of X-Rays

- Spannung: Röntgenstrahlen treten an Anode aus

$E_{electron} = e \cdot U = h \cdot f$

max. freq nimmt mit U linear zu

- mit zunehmender Frequenz und Energie pro Elektron nimmt die Bremsstrahlung zu
- Bremsstrahlung entsteht wenn Elektronen auf niedrigere Schalen im Bohr-Modell fallen

X-Ray Tube Efficiency

$\eta = \frac{\text{Radiated Power}}{\text{Electrical input power}}$

$\eta_{tube} \approx k \cdot U \cdot Z$

$10^5 V$

$10^{-9} \frac{1}{V}$

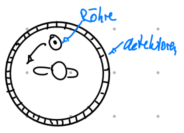
atomic number of anode material

Wolfram: Z = 74

⇒ $\eta < 1\%$

Computer Tomography

- Röhre verschoben und rotiert um Objekt herum
- Generation 4



- Verwendung von Radon Transformation:

$R_f(c) = \int_c f(x,y)\,ds$

↳ Inverse von CT